

# The holostar - a self-consistent model for a compact self-gravitating object

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## Abstract

Anisotropic compact stars were introduced as a new class of solutions to Einstein's classical field equations of general relativity in several recent papers. Some of these solutions possess a two-dimensional membrane of tangential pressure situated at the boundary between the matter-filled interior and the outer vacuum space-time. In this paper the so called holographic solution, in short "holostar", is discussed. The holostar is characterized by the property, that the stress-energy content of its membrane is equal to the gravitational mass of the holostar.

The holostar exhibits properties similar to the properties of black holes. The exterior space-time of the holostar is identical to that of a Schwarzschild black hole, due to Birkhoff's theorem. The holostar possesses an internal temperature, proportional to  $1/\sqrt{r}$ , from which the Hawking temperature law for spherically symmetric black holes can be derived up to a constant factor. The number of zero rest-mass particles within any concentric region of the holostar's interior is proportional to the proper area of its boundary, implying that the holostar is compatible with the holographic principle and the Bekenstein entropy-area bound. In contrast to a black hole, the holostar-metric is static throughout the whole space-time. There are no trapped surfaces, no singularity and no event horizon. Information is not lost. The weak and strong energy conditions are fulfilled everywhere, except for a Planck-size region at the center. Therefore the holostar can serve as an alternative model for a compact self-gravitating object.

Although the holostar is a static solution, it behaves dynamically with respect to the interior motion of its constituent particles. Geodesic motion of massive particles in a large holostar exhibits properties quite similar to what is found in the observable universe: Any material observer moving geodesically will observe an isotropic outward directed Hubble-flow of massive particles from his local frame of reference. The radial

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motion is accelerated, with the proper acceleration falling off over time. The acceleration is due to the interior metric, there is no cosmological constant. The total matter-density  $\rho$ , viewed from the extended Lorentz-frame of a geodesically moving observer, decreases over proper time  $\tau$  with  $\rho \propto 1/\tau^2$ . The radial coordinate position  $r$  of the observer changes proportional to  $\tau$ . The local Hubble value is given by  $H \simeq 1/\tau$ . The observer is immersed in a bath of zero rest-mass particles (photons), whose temperature decreases with  $T \propto 1/\sqrt{\tau}$ , i.e.  $\rho \propto T^4$ . Geodesic motion of photons within the holostar preserves the Planck-distribution. The radial position of an observer can be determined via the local mass-density, the local radiation-temperature or the local Hubble-flow. Using the experimentally determined values for the matter-density of the universe, the Hubble-value and the CMBR-temperature, values of  $r$  between 8.06 and  $9.18 \cdot 10^{60} r_{Pl}$  are calculated, i.e values close to the current radius and age of the universe. Therefore the holographic solution might serve as an alternative model for a universe with anisotropic negative pressure, without need for a cosmological constant.

The holographic solution also admits microscopic self-gravitating objects with a surface area of roughly the Planck-area and zero gravitating mass.

## 1 Introduction:

In a series of recent papers new interest has grown in the problem of finding the most general solution to the spherically symmetric equations of general relativity, including matter. Many of these papers deal with anisotropic matter states.<sup>1</sup> Anisotropic matter - in a spherically symmetric context - is a (new) state of matter, for which the principal pressure components in the radial and tangential direction differ. Note that an anisotropic pressure is fully compatible with spherical symmetry, a fact that appears to have been overlooked by some of the old papers. One of the causes for this newly awakened interest could be the realization, that models with anisotropic pressure appear to have the potential to soften up the conditions under which spherically symmetric collapse necessarily proceeds to a singularity.<sup>2</sup> Another motivation for the renewed interest might be the prospect of the new physics that will have to be developed in order to understand the peculiar properties of matter in a state of highly anisotropic pressure and to determine the conditions according to which such matter-states develop.

In this paper a particularly simple model for a spherically symmetric, self gravitating system with a highly, in fact maximally anisotropic pressure is studied. The model provides some new insights into the physical phenomena of

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<sup>1</sup>Relevant contributions in the recent past (most likely not a complete list of relevant references) can be found in the papers of [4, 6, 8, 9, 10, 11, 12, 14, 13, 16, 17, 18, 19, 20, 21, 22, 29, 34, 36] and the references given therein.

<sup>2</sup>See for example [36], who noted that under certain conditions a finite region near the center necessarily expands outward, if collapse begins from rest.

strongly gravitating systems, such as black holes, the universe and - possibly - even elementary particles.

The paper is divided into three sections. In the following principal section some characteristic properties of the holostar solution are derived. In the next section these properties are compared to the properties of the most fundamental objects of nature that are known so far, i.e. elementary particles, black holes and the universe. The question, whether the holostar can serve as an alternative, unified model for these fundamental objects will be discussed. The paper closes with a discussion and outlook.

## 2 Some characteristic properties of the holographic solution

In this section some characteristic properties of the holostar are derived. As the exterior space-time of the holostar is identical to the known Schwarzschild vacuum solution, only the interior space-time will be covered in detail.

Despite the mathematically simple form of the interior metric, the holostar's interior structure and dynamics turns out to be far from trivial. A remarkable list of properties can be deduced from the interior metric, indicating that the holostar has much in common with a spherically symmetric black hole and with the observable universe.

### 2.1 The holostar metric

The holographic solution is a particular case of a spherically symmetric solution to the equations of general relativity with an interior matter-density proportional to  $1/r^2$ . In units  $c = G = 1$  the metric and (interior) fields are given by [29] as:

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2d\Omega \quad (1)$$

$$B(r) = \frac{1}{A(r)} = \frac{r_0}{r}(1 - \theta(r - r_h)) + \left(1 - \frac{r_+}{r}\right)\theta(r - r_h) \quad (2)$$

$$\rho(r) = \frac{1}{8\pi r^2}(1 - \theta(r - r_h)) \quad (3)$$

$$P_r(r) = -\rho(r) \quad (4)$$

$$P_\theta = \frac{1}{16\pi r_h} \delta(r - r_h) \quad (5)$$

$r_h = r_+ + r_0$  is the position of the membrane,  $r_+ = 2M$  is the gravitational radius of the holostar,  $M$  its gravitating mass and  $r_0$  is a fundamental length. There is evidence [28, 30], that  $r_0$  is comparable to the Planck-length up to a numerical factor of order unity ( $r_0 \approx 2r_{Pl}$ ).

The matter-fields, i.e.  $\rho$ ,  $P_r$  and  $P_\theta$  can be derived from the metric, once the metric of the whole space-time is known.<sup>3</sup>

## 2.2 Proper volume and radial distance

The proper radius, i.e. the proper length of a radial trajectory from the center to radial coordinate position  $r$ , of the holostar scales with  $r^{3/2}$ :

$$l(0, r) = \int_0^r \sqrt{A} dr = \frac{2}{3} r \sqrt{\frac{r}{r_0}} \quad (6)$$

The proper radial distance between the membrane at  $r_h$  and the gravitational radius at  $r_+$  is given by:

$$l(r_+, r_h) = \frac{2}{3} r_+ \sqrt{\frac{r_+}{r_0}} \left( \left(1 + \frac{r_0}{r_+}\right)^{\frac{3}{2}} - 1 \right) \cong r_0 \sqrt{\frac{r_+}{r_0}} = \sqrt{r_+ r_0} \quad (7)$$

The proper volume of the region enclosed by a sphere with proper area  $4\pi r^2$  scales with  $r^{7/2}$ :

$$V(r) = \int_0^r 4\pi r^2 \sqrt{\frac{r}{r_0}} dr = \frac{6}{7} \sqrt{\frac{r}{r_0}} V_{flat} \quad (8)$$

$V_{flat} = (4\pi/3)r^3$  is the volume of the respective sphere in flat space.

Therefore both volume and radial distance in the interior space-time region of the holostar are enhanced over the respective volume or radius of a sphere in flat space by the square-root of the ratio between  $r_h = r_+ + r_0$  and the fundamental distance defined by  $r_0$ .

The proper integral over the mass-density, i.e. the sum over the total constituent matter, scales as the proper radius, i.e. as  $r_h^{3/2}$ . There is evidence [30] that the interior matter should consist of at least one mass-less fermion species.<sup>4</sup>

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<sup>3</sup>See for example [29]

<sup>4</sup>It is quite remarkable and maybe not just a coincidence, that  $r^{3/2}$  is the dimension of a (supersymmetric) fermion field in appropriate units.

## 2.3 Energy-conditions

For a space-time with a diagonal stress-energy tensor,  $T_\mu^\nu = \text{diag}(\rho, -P_1, -P_2, -P_3)$ , the energy conditions can be stated in the following form:

- weak energy condition:  $\rho \geq 0$  and  $\rho + P_i \geq 0$
- strong energy condition:  $\rho + \sum P_i \geq 0$  and  $\rho + P_i \geq 0$
- dominant energy condition:  $\rho \geq |P_i|$

It is easy to see that the holostar fulfills the weak and strong energy conditions at all space-time points, except at  $r = 0$ , where - formally - a negative point mass of roughly Planck-size is situated. The dominant energy condition is fulfilled everywhere except at  $r = 0$  and at the position of the membrane  $r = r_h$ .

According to quantum gravity, the notion of space-time points is ill defined. Space-time loses its smooth manifold structure at small distances. At its fundamental level the geometry of space time should be regarded as discrete [24, 25]. The minimum (quantized) area in quantum gravity is non-zero and roughly equal to the Planck area [3, 33]. Therefore the smallest physically meaningful space-time region will be bounded by a surface of roughly Planck-area. Measurements probing the interior of a minimal space-time region make no sense. A minimal space-time region should be regarded as devoid of any physical (sub-)structure.

If the energy conditions are evaluated with respect to physically meaningful space-time regions<sup>5</sup>, i.e. by integrals over at least Planck-sized regions, the following picture emerges: Due to the negative point mass at the center of the holostar the weak energy condition is violated in the sub Planck size region  $r < r_0$ . However, from the viewpoint of quantum gravity this region should be regarded as inaccessible for any meaningful physical measurement.

I therefore propose to discard the region  $r < r_0$  from the physical picture. This deliberate exclusion of a classically well defined region might appear somewhat conceived. From the viewpoint of quantum gravity it is quite natural. Furthermore, disregarding the region  $r < r_0$  is not inconsistent. In fact, the holographic solution in itself very strongly suggests, that whatever is "located" in the region  $r < r_0$  is irrelevant to the (classical) physics outside of this region, not only from a quantum, but also from a purely classical perspective: If we "cut out" the region  $r < r_0$  from the holostar space-time and identify all space-time points on the sphere  $r = r_0$ , we arrive at exactly the same space-time in the physically relevant region  $r \geq r_0$ , as if the region  $r < r_0$  had been included. The reason for this is, that the gravitational mass of the region

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<sup>5</sup>With "Planck-sized region" a compact space-time region of non-zero, roughly Planck-sized volume bounded by an area of roughly the Planck-area is meant.

$r \leq r_0$  - evaluated classically - is exactly zero. This result is due to the (unphysical) negative point mass at the (unphysical) position  $r = 0$  in the classical solution, which cancels the integral over the mass-density  $\propto 1/r^2$  in the interval  $(0, r_0]$ . Although neither the negative point mass nor the infinite mass-density and pressure at the center of the holostar are acceptable, they don't have any classical effect outside the physically meaningful region  $r > r_0$ .

Thus the holographic solution satisfies the weak (positive) energy condition in any physically meaningful space-time region throughout the whole space-time manifold. The same is true for the strong energy condition.

Can one "mend" the violation of the dominant energy condition in the membrane by a similar argument? Due to the considerable surface pressure of the membrane the dominant energy condition is clearly not satisfied within the membrane. Unfortunately there is no way to fulfill the dominant energy condition by "smoothing" over a Planck-size region<sup>6</sup>, as is possible in case of the weak and strong energy conditions. Therefore the violation of the dominant energy condition by the membrane must be considered as a real physical effect, i.e. a genuine property of the holostar.

Is the violation of the dominant energy condition incompatible with the most basic physical laws? The dominant energy condition can be interpreted as saying, that the speed of energy flow of matter is always less than the speed of light. As the dominant energy condition is violated in the membrane, one must expect some "non-local" behavior of the membrane. Non-locality, however, is a well known property of quantum phenomena. Non-local behavior of quantum systems has been verified experimentally up to macroscopic dimensions.<sup>7</sup> This suggests that the membrane might be a macroscopic quantum phenomenon. In [30] it is proposed, that the membrane should consist of a condensed boson gas at a temperature far below the Bose-temperature of the membrane. In such a case the membrane could be characterized as a single macroscopic quantum state of bosons. One would expect collective non-local behavior from such a state.

## 2.4 "Stress-energy content" of the membrane

One of the outstanding characteristics of the holostar is the property, that the "stress-energy content" of the membrane is equal to the gravitating mass  $M$  of the holostar:

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<sup>6</sup>If the term "Planck-sized region" would only refer to volume, allowing an arbitrary large boundary area, we could construct a cone-shaped region extending from the center of the holostar to the membrane, with arbitrary small solid angle, but huge area and radial extension. In such a cone-shaped region the integrated dominant energy condition would not be violated, if the region extends from the membrane at least half-way to the center.

<sup>7</sup>See for example the spin "entanglement" experiments, "quantum teleportation" and quantum cryptography, just to name a few phenomena, that depend on the non-local behavior of quantum systems. Some of these phenomena, such as quantum-cryptography, are even being put into technical use.

For any spherically symmetric solution to the equations of general relativity the total gravitational mass  $M(r)$  within a concentric space-time region bounded by  $r$  is given by the following mass function:

$$M(r) = \int_0^r \rho \widetilde{dV} = \int_0^r \rho 4\pi r^2 dr \quad (9)$$

$M(r)$  is the integral of the mass-density  $\rho$  over the (improper) volume element  $\widetilde{dV} = 4\pi r^2 dr$ , i.e. over a spherical shell with radial coordinate extension  $dr$  situated at radial coordinate value  $r$ . Note, that in a space-time with  $AB = 1$  the so called "improper" integral over the interior mass-density appears just the right way to evaluate the asymptotic gravitational mass  $M$ : The gravitational mass can be thought to be the genuine sum (i.e. the proper integral) over the constituent matter, corrected by the gravitational red-shift. The proper volume element is given by  $dV = 4\pi r^2 \sqrt{A} dr$ . The red-shift factor for an asymptotic observer situated at infinity, with  $B(\infty) = 1$ , is given by  $\sqrt{B}$ . Therefore the proper integral over the constituent matter, red-shift corrected with respect to an observer at spatial infinity, is given by:  $M = \int \rho \sqrt{AB} 4\pi r^2 dr$ . This is equal to the improper integral in equation (9), whenever  $AB = 1$ .

If the energy-content of the membrane is calculated by the same procedure, replacing  $\rho$  with the two principal non-zero pressure components  $P_\theta = P_\varphi$  in the membrane, one gets:

$$\int_0^\infty (2P_\theta) 4\pi r^2 dr = \frac{r_h}{2} = M + \frac{r_0}{2} \cong M \quad (10)$$

Note, that the tangential pressure in the membrane of a holostar,  $P_\theta = 1/(16\pi(2M+r_0))$ , is (almost) exactly equal to the tangential pressure attributed to the event horizon of a spherically symmetric black hole by the membrane paradigm [31, 40],  $P_\theta = 1/(32\pi M)$ .

The integral over the mass-density (or over some particular pressure components) might not be considered as a very satisfactory means to determine the total gravitational mass of a self gravitating system. Neither the mass-density nor the principal pressures have a coordinate independent meaning: They transform like the components of a tensor, not as scalars.

In this respect it is quite remarkable, that the gravitating mass of the holostar can be derived from the integral over the trace of the stress-energy tensor  $T = T^\mu_\mu$ . In fact, the integral over  $T$  is exactly equal to  $M$  if the negative point mass  $M_0 = -r_0/2$  at the center is included, or - which is the preferred procedure - if the integration is performed from  $r \in [r_0, \infty]$ , as was suggested in the previous section.

$$\int_0^\infty T 4\pi r^2 dr = \int_0^\infty (\rho - P_r - 2P_\theta) 4\pi r^2 dr = \frac{r_h}{2} + M_0 = M \quad (11)$$

Contrary to the mass-density and/or the pressure,  $T$  is a Lorentz-scalar and therefore has a definite coordinate-independent meaning at any space-time point.

It is quite interesting that the gravitating mass of the holostar also can be derived from the so called Tolman-mass  $M_{Tol}$  of the space-time:

$$M_{Tol} = \int_0^\infty (\rho + P_r + 2P_\theta) \sqrt{-g} d^3x = \frac{r_h}{2} + M_0 = M \quad (12)$$

The Tolman mass is often referred to as the "active gravitational mass" of a system. The motion of particles in a wide class of exact solutions to the equations of general relativity indicate that the sum of the matter density and the three principle pressures can be interpreted as the "true source" of the gravitational field, or rather of the field's action, its gravitational attraction.<sup>8</sup>

Also note, that the rr- and tt-components of the Ricci-tensor are zero everywhere, except for a  $\delta$ -functional at the position of the membrane:

$$R_0^0 = R_1^1 = -\frac{1}{2r_h} \delta = -\frac{1}{4M + 2r_0} \delta$$

## 2.5 The equations of geodesic motion

In this section the equations for the geodesic motion of particles within the holostar are set up. Keep in mind that the results of the analysis of pure geodesic motion have to be interpreted with caution, as pure geodesic motion is unrealistic in the interior of a holostar. In general it is not possible to neglect the pressure-forces totally. In fact, as will be shown later, it is quite improbable that the motion of massive particles will be geodesic throughout the holostar's whole interior space-time. Nevertheless, the analysis of pure geodesic motion, especially for photons, is a valuable tool to discover the properties of the interior space-time.

Disregarding pressure effects, the interior motion of massive particles or photons can be described by an effective potential. The geodesic equations of motion for a general spherically symmetric space-time, expressed in terms of the "geometric" constants of the motion  $r_i$  and  $\beta_i$  were given in [29]:

$$\beta_r^2(r) + V_{eff}(r) = 1 \quad (13)$$

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<sup>8</sup>Poisson's equation for the local relative gravitational acceleration  $g$  is given by  $\nabla \cdot \mathbf{g} = -R_{00} = -4\pi G(\rho + \sum P_i)$  in local Minkowski-coordinates. Therefore  $\rho$  and the three principal pressures appear as source-terms in Poisson's equation for  $g$ , i.e. can be considered as active gravitational mass densities.

$$V_{eff}(r) = \frac{B(r)}{B(r_i)} \left( 1 - \beta_i^2 \left( 1 - \frac{r_i^2}{r^2} \right) \right) \quad (14)$$

$$\beta_{\perp}^2(r) = \frac{B(r)}{B(r_i)} \frac{r_i^2}{r^2} \beta_i^2 \quad (15)$$

$r_i$  is the interior turning point of the motion,  $\beta_r$  is the radial velocity, expressed as fraction to the local velocity of light in the radial direction,  $\beta_{\perp}$  is the tangential velocity, expressed as fraction to the local velocity of light in the tangential direction. As  $\partial r, \partial \varphi$  and  $\partial \theta$  are orthogonal,  $\beta^2 = \beta_r^2 + \beta_{\perp}^2$ . The quantities  $\beta^2, \beta_r^2$  and  $\beta_{\perp}^2$  all lie in the interval  $[0, 1]$ .

$\beta_i = \beta(r_i) = \beta_{\perp}(r_i)$  is the velocity of the particle at the turning point of the motion,  $r_i$ .  $\beta_r(r_i) = 0$ , therefore  $\beta_i$  is purely tangential at  $r_i$ . For photons  $\beta_i = 1$ . Pure radial motion for photons is only possible, when  $r_i = 0$ . In this case  $V_{eff} = 0$ . Therefore the purely radial motion of photons can be considered as "force-free".<sup>9</sup>

In order to integrate the geodesic equations of motion, the following relations are required:

$$\beta_r(r) = \frac{dr}{dt} / \sqrt{\frac{B}{A}} \quad (16)$$

$$\beta_{\perp}(r) = \frac{r d\varphi}{dt} / \sqrt{B} \quad (17)$$

$t$  is the time measured by the asymptotic observer at spatial infinity. If the equations of motion are to be solved with respect to the proper time  $\tau$  of the particle (this is only reasonable for massive particles with  $\beta < 1$ ), the following relation is useful:

$$dt = d\tau \frac{\gamma_i \sqrt{B(r_i)}}{B} \quad (18)$$

$\gamma^2 = 1/(1 - \beta^2)$  is the special relativistic  $\gamma$ -factor and  $\gamma_i = \gamma(r_i)$ , the local  $\gamma$ -factor of the particle at the turning point of the motion.

Within the holostar  $B = r_0/r$ . Therefore the equations of motion reduce to the following set of simple equations:

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<sup>9</sup>The region  $r < r_0$  should be considered as inaccessible, which means that pure radial motions for photons is impossible. Aiming the photons precisely at  $r = 0$  also conflicts with the quantum mechanical uncertainty postulate. The photons will therefore always be subject to an effective potential with  $r_i > r_0$ , i.e. an effective potential that is not constant. However, whenever the photon has moved an appreciable distance from its turning point of the motion, i.e. whenever  $r \gg r_i$ , the effective potential is nearly zero. Therefore non-radial motion of photons can be regarded as nearly "force-free", whenever  $r \gg r_i$ .

$$\beta_r^2(r) = 1 - \frac{r_i}{r} \left( 1 - \beta_i^2 \left( 1 - \frac{r_i^2}{r^2} \right) \right) = \left( \frac{dr}{dt} \frac{r}{r_0} \right)^2 \quad (19)$$

$$\beta_\perp^2(r) = \beta_i^2 \left( \frac{r_i}{r} \right)^3 = \left( \frac{rd\varphi}{dt} \right)^2 \frac{r}{r_0} \quad (20)$$

The equation for  $\varphi$  can be expressed as a function of radial position  $r$ , instead of time:

$$\frac{d\varphi}{dr} = \frac{1}{r^2 \beta_r(r)} \sqrt{\frac{r_i}{r_0}} r_i \beta_i \quad (21)$$

Equation (21) determines the orbit of the particle in the spatial geometry. It is not difficult to integrate. For  $r \gg r_i$  the radial velocity  $\beta_r(r)$  is nearly unity, independent of the nature of the particle and of its velocity at the turning point of the motion,  $\beta_i$ . In the region  $r \gg r_i$  we find  $d\varphi \propto dr/r^2$ , so that the angle remains nearly unchanged. This implies, that the number of revolutions of an interior particle around the holostar's center is limited. The radial coordinate position  $r_1$  from which an interior particle can at most perform one more revolution is given by  $\varphi(\infty) - \varphi(r_1) < 2\pi$ . Expressing  $r_1$  in multiples of  $r_i$  yields:

$$\frac{r_1}{r_i} > \beta_i \sqrt{\frac{r_i}{r_0}} \frac{1}{2\pi} \quad (22)$$

The total number of revolutions of an arbitrary particle, emitted with tangential velocity component  $\beta_i$  from radial coordinate position  $r_i$  is very accurately estimated by the number of revolutions in an infinitely extended holostar:

$$N_{rev} \simeq \frac{1}{2\pi} \int_{r_i}^{\infty} \frac{d\varphi}{dr} = \beta_i \sqrt{\frac{r_i}{r_0}} \frac{1}{2\pi} \int_0^1 \frac{dx}{\sqrt{1-x(1-\beta_i^2(1-x^2))}} \quad (23)$$

The definite integral in equation (23) only depends on  $\beta_i$ . Its value lies between 1.4022 and 2, the lower value for  $\beta_i = 1$ , i.e. for photons<sup>10</sup>, and the higher value for  $\beta_i = 0$ , i.e. for pure radial motion of massive particles.<sup>11</sup> The integral is a monotonically decreasing function of  $\beta_i^2$ . From equation (23) it is quite obvious that particles emitted from the central region of the holostar will cover only a small angular portion of the interior holostar space-time.

In the following sections we are mainly interested in the radial part of the motion of the particles.

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<sup>10</sup>The exact value of the definite integral for  $\beta_i = 1$  is given by  $\frac{\sqrt{\pi}}{3} \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}$

<sup>11</sup>In this case the number of revolutions is zero, as  $\beta_i = 0$

From equation (14) it can be seen, that whenever  $B(r)$  is monotonically decreasing, the effective potential is monotonically decreasing as well, independent of the constants of the motion,  $r_i$  and  $\beta_i$ . Within the holostar  $B(r) = r_0/r$ , therefore the interior effective potential decreases monotonically from the center at  $r = 0$  to the boundary at  $r = r_h$ , which implies that the radial velocity of an outmoving particle increases steadily. The motion appears accelerated from the viewpoint of an exterior asymptotic observer. The perceived acceleration decreases over time, as the effective potential becomes very flat for  $r \gg r_i$ .

For all possible values of  $r_i$  and  $\beta_i$  the position of the membrane is a local minimum of the effective potential. Therefore no motion is possible that remains completely within the holostar's interior. Any interior particle has two options: Either it oscillates between the interior and the exterior space-time or it passes over the angular momentum barrier situated in the exterior space-time (or tunnels through) and escapes to infinity.

In order to discuss the general features of the radial motion it is not necessary to solve the equations exactly. For most purposes we can rely on approximations.

For photons the radial equation of motion is relatively simple:

$$\beta_r(r) = \sqrt{1 - \left(\frac{r_i}{r}\right)^3} = \frac{dr}{dt} \frac{r}{r_0} \quad (24)$$

An exact integration requires elliptic functions. For  $r \gg r_i$  the term  $(r_i/r)^3$  under the root can be neglected, so that the solution can be expressed in terms of elementary functions.

For massive particles the general equation (19) is very much simplified for pure radial motion, i.e.  $\beta_i = 0$ :

$$\beta_r(r) = \sqrt{1 - \frac{r_i}{r}} = \frac{dr}{dt} \frac{r}{r_0} \quad (25)$$

This equation can be integrated with elementary mathematical functions. The general equation of motion (19) requires elliptic integrals. However, in the general case of the motion of a massive particle ( $\beta_i \neq 0$  and  $\beta_i^2 < 1$ ) the equation for the radial velocity component (19) can be approximated as follows for large values of the radial coordinate coordinate,  $r \gg r_i$ :

$$\beta_r^2(r) \simeq 1 - \frac{r_i}{r} (1 - \beta_i^2) = 1 - \frac{r_i}{r} \frac{1}{\gamma_i^2} \quad (26)$$

Whenever  $r \gg r_i$  the solution to the general radial equation of motion for a massive particle is very well approximated by the much simpler, analytic solution for pure radial motion of a massive particle given by equation (25). One only has to replace  $r_i$  by  $r_i/\gamma_i^2$ . The radial component of the motion of a particle

emitted with arbitrary  $\beta_i$  from  $r_i$  is nearly indistinguishable from the motion of a massive particle that started out "at rest" in a purely radial direction from a somewhat smaller "fictitious" radial coordinate value  $\tilde{r}_i = r_i/\gamma_i^2$ .

## 2.6 Bound versus unbound motion

In this section I discuss some qualitative features of the motion of massive particles and photons in the holostar's gravitational field.

An interior particle is bound, if the effective potential at the interior turning point of the motion,  $r_i$ , is equal to the effective exterior potential at an exterior turning point of the motion,  $r_e$ . The effective potential has been normalized such, that at any turning point of the motion  $V_{eff}(r) = 1$ . A necessary (and sufficient) condition for bound motion then is, that the equation

$$V_{eff}(r_e) = 1 \quad (27)$$

has a real solution  $r_e \geq r_h$  in the exterior (Schwarzschild) space-time.

### 2.6.1 Bound motion of massive particles

For pure radial motion and particles with non-zero rest-mass equation (27) is easy to solve. We find the following relation between the exterior and interior turning points of the motion:

$$r_e = \frac{r_+}{1 - \frac{r_0}{r_i}} \quad (28)$$

Equation (28) indicates, that for massive particles bound orbits are only possible if  $r_i > r_0$ . If the massive particles have angular momentum, the turning point of the motion,  $r_i$ , must be somewhat larger than  $r_0$ , if the particles are to be bound. Angular motion therefore increases the central "unbound" region. The number of massive particles in the "unbound" central region, however, is very small. In [30] it will be shown, that the number of (ultra-relativistic) constituent particles within a concentric interior region of the holostar in thermal equilibrium is proportional to the boundary surface of the region with  $N \approx (r/r_0)^2$ .

Let us assume that a massive particle has an interior turning point of the motion far away from the central region, i.e.  $r_i \gg r_0$ . Equation (28) then implies, that for such a particle the exterior turning point of the motion will lie only a few Planck-distances outside the membrane. This can be seen as follows:

If a massive particle is to venture an appreciable distance away from the membrane, the factor  $1 - r_0/r_i$  on the right hand side of equation (28) must

deviate appreciably from unity. This is only possible if  $r_0 \approx r_i$ . In the case  $r_i \gg r_0$  equation (28) gives a value for the exterior turning point of the motion,  $r_e$ , that is very close to the gravitational radius of the holostar. Any particle emitted from the region  $r_i \gg r_0$  will barely get past the membrane. Even particles whose turning point of the motion is as close as two fundamental length units from the center of the holostar, i.e.  $r_i = 2r_0$ , have an exterior turning point of the motion situated just one gravitational radius outside of the membrane at  $r_e = 2r_+$ .

The above analysis demonstrates, that only very few, if any, massive particles can escape the holostar's gravitational field on a classical geodesic trajectory. As has been remarked before, an escape to infinity is only possible for massive particles (with zero angular momentum) emanating from a sub Planck-size region of the center ( $r_i < r_0$ ). It is quite unlikely that this region will contain more than one particle.

In the case of angular motion the picture becomes more complicated. In general the equation  $V(r_e) = 1$  is a cubic equation in  $r_e$ :

$$B(r_i) = \frac{r_0}{r_i} = \left(1 - \frac{r_+}{r_e}\right) \left(1 - \beta_i^2 \left(1 - \frac{r_i^2}{r_e^2}\right)\right) \quad (29)$$

It is possible to solve this equation by elementary methods. The formula are quite complicated. The general picture is the following: For any given  $r_i$  the particle becomes "less bound"<sup>12</sup>, the higher the value of  $\beta_i^2$  at the interior turning point gets. Particles with interior turning point of the motion close to the center are "less bound" than particles with interior turning point close to the boundary. For sufficiently small  $r_i$  bound motion is not possible, whenever a certain value of  $\beta_i^2$  is exceeded.

For particles with  $r_i \ll r_e$  equation (29) is simplified, as  $r_i^2/r_e^2$  can be neglected. We find

$$\frac{r_0}{r_i} \approx \left(1 - \frac{r_+}{r_e}\right)(1 - \beta_i^2) = \left(1 - \frac{r_+}{r_e}\right) \frac{1}{\gamma_i^2} \quad (30)$$

which allows us to determine  $r_e$  to a fairly good approximation:

$$r_e \approx \frac{r_+}{1 - \frac{r_0 \gamma_i^2}{r_i}} \quad (31)$$

The motion is bound, as long as  $r_i > \gamma_i^2 r_0$ . This inequality seems to indicate, that any massive particle is able to escape the holostar in principle, as long as  $\gamma_i$  is high enough. This however, is not true:

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<sup>12</sup>i.e.  $r_e$  becomes larger than the value given in equation (28)

The geodesic motion of massive particles and photons is described by an effective potential, which is a function of  $r_i$  and  $\beta_i$ . From equation (14) it can be seen, that the exterior effective potential for fixed  $r_i$  is a monotonically decreasing function of  $\beta_i^2$ , i.e.  $V_{eff}(r, \beta_i^2) \geq V_{eff}(r, 1)$ . This is true for any radial position  $r$ . Therefore the exterior effective potential of any particle with fixed  $r_i$  and arbitrary  $\beta_i$  is bounded from below by the effective potential of a photon. This implies that whenever a photon possesses an exterior turning point of the motion, any particle with  $\beta_i^2 < 1$  must have an exterior turning point as well, somewhat closer to the holostar's membrane. Therefore escape to infinity from any interior position  $r_i$  for an arbitrary rest-mass particle is only possible, if a photon can escape to infinity from  $r_i$ .

In the following section the conditions for the unbound motion of photons are analyzed.

### 2.6.2 Unbound motion of photons

The effective potential for photons at spatial infinity is zero. Therefore in principal any photon has the chance to escape the holostar permanently. Usually this doesn't happen, due to the photon's angular momentum.

Disregarding the quantum-mechanical tunnel-effect, the fate of a photon, i.e. whether it remains bound in the gravitational field of the holostar or whether it escapes permanently on a classical trajectory to infinity, is determined at the angular momentum barrier, which is situated in the exterior space-time at  $r = 3r_+/2$ . The exterior effective potential for photons possesses a local maximum at this position. If the effective potential for a photon at  $r = 3r_+/2$  is less than 1, i.e.  $V_{eff}(3r_+/2) < 1$ , the photon has a non-zero radial velocity at the maximum of the angular momentum barrier. Escape is classically inevitable. The condition for (classical) escape for photons is thus given by:

$$\frac{r_i}{r_0} \leq 3 \left( \frac{r_+}{2r_0} \right)^{\frac{2}{3}} \quad (32)$$

Any photon emitted from the region defined by equation (32) will escape, if its motion is purely geodesic.

On the other hand, any photon whose internal turning point of the motion lies outside the "photon-escape region" defined by equation (32) will be turned back at the angular momentum barrier and therefore is bound. Likewise, all massive particles outside the photon escape region must be bound as well, because the exterior effective potential of a massive particle is always larger than that of a photon with the same  $r_i$ , i.e. the angular momentum barrier for a massive particle is always higher than that of a photon emitted from the same  $r_i$ . Therefore every particle with interior turning point of the motion outside the "photon escape region" will be turned back by the angular momentum barrier.

For large holostars, the region of classical escape for photons becomes arbitrarily small with respect to the holostar's overall size. A holostar of the size of the sun with  $r_+/r_0 \approx 10^{38}$  has an "unbound" interior region of  $r_i \leq 4.10^{25}r_0 \approx 0.5\text{ nm}$ . The radial extension of the "photon escape region" is 13 orders of magnitude less than the holostar's gravitational radius. The gravitational mass of this region is negligible compared to the total gravitational mass of the holostar.<sup>13</sup>

### 2.6.3 Unbound motion of massive particles

For particles with non-zero rest-mass the analysis is very much simplified, if the effect of the angular momentum barrier is neglected.<sup>14</sup> Massive particles generally have an effective potential at spatial infinity larger than zero. A necessary, but not sufficient condition for a massive particle to be unbound is, that the effective potential at spatial infinity be less than 1. This condition translates to:

$$r_i < \frac{r_0}{1 - \beta_i^2} = r_0 \gamma_i^2 \quad (33)$$

According to equation (33) escape on a classical geodesic trajectory for a massive particle is only possible from a region a few Planck-lengths around the center, unless the particle is highly relativistic. For example, massive particles with  $\beta_i^2 = 0.5$  can only be unbound, if they originate from the region  $r_i < 2r_0$ . If the region of escape for massive particles is to be macroscopic, the proper tangential velocity  $\beta_i^2$  at the turning point of the motion must be phenomenally close to the local speed of light. Note however that in such a case there usually is an angular momentum barrier in the exterior space-time (see the discussion in the previous section).

## 2.7 An upper bound for the particle flux to infinity

The lifetime of a black hole, due to Hawking evaporation, is proportional to  $M^3$ . Hawking radiation is independent of the interior structure of a black hole. It depends solely on the exterior metric up to the event horizon. As the exterior space-times of the holostar and a black hole are identical (disregarding the Planck-size region between gravitational radius and membrane) the estimated lifetime of the holostar, due to loss of interior particles, should not significantly deviate from the Hawking result.

<sup>13</sup>Quite interestingly, for a holostar of the mass of the universe ( $r \approx 10^{61}r_{Pl}$ ), the temperature at the radius of the photon-escape region is  $T \approx 2.7 \cdot 10^{10}K \approx 2.3MeV$ , which is quite close to the temperature of nucleosynthesis.

<sup>14</sup>For most combinations of  $r_i$  with  $\beta_i^2 < 1$ , the angular momentum barrier hasn't a significant effect on the question, whether a the particle is bound or unbound.

An upper bound for the flux of particles from the interior of the holostar to infinity can be derived by the following, albeit very crude argument:

Under the - as we will later see, unrealistic - assumption, that the effects of the negative radial pressure can be neglected, the particles move on geodesics and the results derived in the previous sections can be applied.

For large holostars and ignoring the pressure the particle flux to infinity will be dominated by photons or other zero rest-mass particles, such as neutrinos, emanating from the "photon escape region"  $r_i < Cr_+^{2/3}r_0^{1/3}$  defined by equation (32).

The gravitational mass  $\Delta M$  of this region, viewed by an asymptotic observer at infinity, is proportional to  $r_+^{2/3}r_0^{1/3}$ .

The exterior asymptotic time  $\Delta t$  for a photon to travel from  $r_i$  to the membrane at  $r_h \simeq r_+$  is given by:

$$\Delta t = \int_{r_i}^{r_+} \sqrt{\frac{A}{B}} \frac{dr}{\beta_r} = \int_{r_i}^{r_+} \frac{r}{r_0} \frac{dr}{\sqrt{1 - \left(\frac{r_i}{r}\right)^3}}$$

For a large holostar with  $r_i \ll r_+$  the integral can be approximated by:

$$\int_{r_i}^{r_+} \frac{r}{r_0} \frac{dr}{\sqrt{1 - \left(\frac{r_i}{r}\right)^3}} \approx \int_0^{r_+} \frac{r}{r_0} dr = \frac{r_+^2}{2r_0}$$

Note, that the time of travel from the membrane,  $r_h$ , to a position  $r$  well outside the gravitational radius of the holostar is of order  $r - r_h$ , i.e. very much shorter than the time of travel from the center of the holostar to the membrane, which is proportional to  $r_h^2$ .

Under the assumption that the continuous particle flux to infinity is comparable to the time average of the - rather conceived - process, in which the whole "photon escape region" is moved in one bunch from the center of the holostar to its surface, one finds the following estimate for the mass-energy-flux to infinity for a large holostar:

$$\frac{\Delta M}{\Delta t} \propto \left(\frac{r_0}{r_+}\right)^{4/3} \propto \left(\frac{\sqrt{h}}{M}\right)^{4/3} \quad (34)$$

This flux is larger than the flux of Hawking radiation, for which the following relation holds:

$$\frac{dM}{dt} \propto \left(\frac{\sqrt{h}}{M}\right)^2$$

However, the pressure effect has not been taken into account in equation (34). As will be shown in the following sections, the pressure reduces the photon flux two-fold: First it reduces the local energy of the outward moving photons, so that less energy is transported to infinity. Second, if the local energy of the individual photons is reduced with respect to pure geodesic motion, the chances of classical escape for a photon are dramatically reduced, because most photons will not have enough "energy" to escape when they finally reach the pressure-free region beyond the membrane.

The first effect reduces the energy of the photon flux by a factor  $r_+^{-1/3}$ , as can be derived from the results of the following section. This tightens the bound given in equation (34) to  $dM/dt \propto 1/M^{5/3}$ .

The second effect will effectively switch off the flux of photons. As will be shown later, the energy of an ensemble of photons moving radially outward or inward changes in such a way, that the ensemble's local energy density is always proportional to the local energy density of the interior matter it encounters along its way. Therefore an ensemble of photons "coming from the interior", having reached the radial position  $r_h$  of the membrane, will be indistinguishable from the photons present at the membrane. The majority of the photons at the membrane, however, will have a turning point of the motion close to  $r_h$ , meaning that escape on a classical trajectory is impossible. Therefore, whenever photons coming from the interior have reached the surface of the holostar,  $r_h$ , their energy will be so low, that the vast majority of the photons are bound. Classically it appears as if no photon will be able to escape from the holostar.

Massive particles which have high velocities at their interior turning points of the motion behave like photons. Therefore the discussion of the previous paragraph applies to those particles as well. Highly relativistic massive particles will not be able to carry a significant amount of mass-energy to infinity. For massive particles escape to infinity is only possible, if the motion starts out from a region within one (or two) fundamental lengths of the center (see equation (33)). But this region contains only very few particles, if any at all.

The holostar therefore must be regarded as classically stable, just as a black hole. Once in a while, however, a particle undergoing random thermal motion close to the surface might acquire sufficient energy in order to escape or tunnel through the angular momentum barrier. Furthermore there are the tidal forces in the exterior space-time, giving rise to "normal" Hawking evaporation.

Taking the pressure-effects into account, the mass-energy flux to infinity of the holographic solution will be quite comparable to the mass-energy flux due to Hawking-evaporation of a black hole. The exponent  $x$  in the energy-flux equation  $dM/dt \propto 1/M^x$  will lie somewhere between  $5/3$  and  $2$ , presumably quite close, if not equal to  $2$ .

Even the very crude upper bound of equation (34) yields quite long lifetimes. For a holostar with the mass of the sun, the evaporation time due to equation (34) is still much larger than the age of the universe ( $T \approx 10^{44}s$ ).

## 2.8 Pressure effects and self-consistency

In this section the effect of the pressure on the internal motion of particles within the holostar is studied. I will demonstrate that the negative, purely radial pressure, equal in magnitude to the mass-density, is an essential property of the holostar, if it is to be a self-consistent static solution.

Let us consider the radial movement (outward or inward) of a spherical shell of particles with a proper thickness  $\delta l = \sqrt{r/r_0} \delta r$ , situated at radial coordinate position  $r$  within the holostar. The shell has a proper volume of  $\delta V = 4\pi r^2 \delta l$  and a total local energy content of  $\rho \delta V = \delta l/2$ .

In this section the analysis will be restricted to zero rest-mass particles, referred to as photons in the following discussion. In the geometric optics approximation photons move along null-geodesic trajectories. Note that the pressure will have an effect on the local energy of the photon. However, as the local speed of light is independent of the photon's energy, the pressure will not be able to change the geometry of a photon trajectory, i.e. the values of  $r(t), \theta(t), \varphi(t)$  as determined from the equations for a null-geodesic trajectory.

For pure radial motion there can be no cross-overs, i.e. no particle can leave the region defined by the two concentric boundary surfaces of the shell.

The equation of motion for a null-geodesic in the interior of the holostar is given by:

$$\frac{dr}{dt} = \frac{r_0}{r} \sqrt{1 - \frac{r_i^3}{r^3}} \quad (35)$$

For  $r \gg r_i$  the square-root factor is nearly one. Therefore whenever the photon has reached a radial position  $r \approx 10r_i$ , a negligible error is made by setting  $r_i = 0$ , which corresponds to pure radial motion.

Equation (35) with  $r_i = 0$  has the solution:

$$r(t) = \sqrt{2r_0 t - r^2(0)} \quad (36)$$

The radial distance between two photons, one travelling on the inner boundary of the shell, starting out at  $r(0) = r_i$ , one travelling on its outer boundary, starting out from  $r(0) = r_i + \delta r_i$ , is given by:

$$\delta r(t) = \sqrt{2r_0 t - (r_i + \delta r_i)^2} - \sqrt{2r_0 t - r_i^2} \quad (37)$$

If  $r(t) \gg r_i$ , Taylor-expansion of the square-root yields:

$$\delta r \approx \delta r_i \frac{r_i}{r} \quad (38)$$

In terms of the proper thickness  $\delta l = \delta r \sqrt{A}$  of the shell we find the following relation:

$$\frac{\delta l}{\delta l_i} = \sqrt{\frac{r_i}{r}} \quad (39)$$

Therefore, if the shell moves radially with the local speed of light, its proper thickness changes according to an inverse square root law.

Whenever the proper radial thickness changes during the movement of the shell, work will be done against the negative radial pressure. The rate of change in proper thickness  $d(\delta l(r))$  per radial coordinate displacement  $dr$  of the shell is given by:

$$d(\delta l) = -\delta l_i \sqrt{r_i} \frac{dr}{2r^{3/2}} \quad (40)$$

Due to the anisotropic pressure (the two tangential pressure components are zero) any change in volume along the tangential direction will have no effect on the total energy. The purely radial pressure has only an effect on the energy of the shell, if the shell expands or contracts in the radial direction. The work done by the negative radial pressure therefore is given by:

$$dE = -P_r 4\pi r^2 d(\delta l) = -\delta l_i \sqrt{r_i} \frac{dr}{4r^{3/2}} \quad (41)$$

Because the radial pressure is negative, the total energy of the shell is reduced when the shell is compressed along the radial direction.

The total pressure-induced local energy change of the shell, when it is moved outward from radial coordinate position  $r_i$  to another position  $r \gg r_i$  is given by the following integral:

$$\Delta E = \int_{r_i}^r dE = \frac{\delta l_i}{2} \left( \sqrt{\frac{r_i}{r}} - 1 \right) \quad (42)$$

But  $\delta l_i/2$  is nothing else than the original total local energy of the shell,  $\delta E_i$ . Therefore we find the following expression for the final energy of the shell:

$$\delta E = \delta E_i + \Delta E = \delta E_i \sqrt{\frac{r_i}{r}} \quad (43)$$

This result could also have been obtained by assuming the ideal gas law  $\delta E \propto P_r \delta V$  with  $\delta V = 4\pi r^2 \delta l$ .

The total energy density in the shell therefore changes according to the following expression:

$$\rho(r) = \frac{\delta E}{\delta V} = \frac{\delta E_i}{4\pi r^2 \delta l_i} = \frac{\delta E_i}{\delta V_i} \frac{r_i^2}{r^2} = \rho(r_i) \frac{r_i^2}{r^2} \quad (44)$$

We have recovered the inverse square law for the mass-density. The holographic solution has a remarkable self-consistency. Any spherical shell carrying a fraction of the total local energy, moving inward or outward with the local velocity of light, changes its energy due to the negative radial pressure in such a way, that the local energy density of the shell at any radial position  $r$  always remains exactly proportional to the actual local energy density  $\rho(r)$  of the (static) holographic solution.

This feature is essential, if the holographic solution is to be a self consistent (quasi-) static solution.<sup>15</sup> A holostar evidently contains matter. A fraction of this matter will consist of non-zero rest-mass particles. These particles will undergo random thermal motion. Furthermore, due to Hawking evaporation or accretion processes there might be a small outward or inward directed net-flux of mass-energy between the interior central core region and the boundary. Even if there is no net mass-energy flux, the different particle species might have non-zero fluxes, as long as all individual fluxes add up to zero. If the internal motion (thermal or directed) takes place in such a way, that the local mass-density of the holostar ( $\rho \propto 1/r^2$ ) is significantly changed from the inverse square law in a time scale shorter than the Hawking evaporation time-scale, the holostar cannot be considered (quasi-) static.

Movement of an interior particle at the speed of light from  $r_i \simeq r_0$  to the membrane at  $r = r_h$  takes an exterior time  $t \simeq r_h^2/(2r_0) \propto M^2$ . The movement of a particle from  $r_i = r_h/2$  to the membrane is not much quicker:  $t \simeq 3r_h^2/(8r_0)$ . In any case the time to move through the holostar's interior is much less than the Hawking evaporation time-scale, which scales as  $M^3$ . Therefore a necessary condition for the holostar to be a self-consistent, quasi-static solution is, that the radial movement of a shell of zero rest mass particles should not disrupt the local (static) mass-density.

More generally any local mass-energy fluxes should take place in such a way, that the overall structure of the holostar is not destroyed and that local mass-energy fluxes aren't magnified to an unacceptable level at large scales.

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<sup>15</sup>The term quasi-static is used, because internal motion within the holostar is possible, as long as there is no net mass-energy flux in a particular direction. A radially directed outward flux of (a fraction of the) interior matter is possible, if this flux is compensated by an equivalent inward flux. Note that the outward and inward flowing matter must not necessarily be of the same kind. If the outward flowing matter consists of massive particles with a finite life-time, the inward flowing matter is expected to carry a higher fraction of the decay products, which will be lighter, possibly zero rest-mass particles.

## 2.9 Number of interior particles and holographic principle

The self-consistency argument given above leads to some interesting predictions. Let us assume that the outmoving shell consists of essentially non-interacting zero rest-mass particles, for example photons or neutrinos (ideal relativistic gas assumption). For a large holostar the mass-density in its outer regions will become arbitrarily low. Therefore the ideal gas approximation should be a reasonable assumption for large holostars. Under this assumption the number of particles in the shell should remain constant.<sup>16</sup> This allows us to determine the number density of zero rest-mass particles within the holostar up to a constant factor:

Let  $N_i$  be the number of particles in the shell at the position  $r_i$ . Then the number density  $n(r)$  of particles in the shell, as it moves inward or outward, is given by the respective change of the shell's volume:

$$n(r) = \frac{N_i}{\delta V} = \frac{N_i}{4\pi r^2 \delta l} = \frac{N_i}{4\pi \sqrt{r_i} \delta l_i} \frac{1}{r^{\frac{3}{2}}} = n(r_i) \left(\frac{r_i}{r}\right)^{\frac{3}{2}} \quad (45)$$

Under the assumption that the interior matter-distribution of the holostar is (quasi-) static, and that the local composition of the matter at any particular  $r$ -position doesn't change with time, self-consistency requires that the number density of zero rest-mass particles per proper volume should be proportional to the number density predicted by equation (45).

The total number of zero rest-mass particles in a holostar will then be given by the proper volume integral of the number density (45) over the whole interior volume:

$$N \propto \int_0^{r_h} \frac{dV}{r^{3/2}} = \int_0^{r_h} \frac{4\pi r^2 \sqrt{\frac{r}{r_0}} dr}{r^{3/2}} \propto r_h^2 \propto A_h \quad (46)$$

We arrive at the remarkable result, that the total number of zero rest-mass particles within a holostar should be proportional to the area of its boundary surface,  $A_h$ . The same result holds for any concentric sphere within the holostar's interior. This is an - albeit still very tentative - indication, that the holographic principle is valid in classical general relativity, at least for large compact self gravitating objects.

Under the assumption that the region  $r < r_0/2 \approx \sqrt{\hbar}$  can contain at most one particle (see the discussion in [28]), we find:

$$N = \left(\frac{r_h}{r_0/2}\right)^2 \approx \frac{r_h^2}{\hbar} = \frac{A_h}{4\pi\hbar} \quad (47)$$

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<sup>16</sup>In general relativity the geometric optics approximation implies the conservation of photon number.

## 2.10 Local radiation temperature and the Hawking temperature

Under the assumption that the mass-energy density of the holostar is dominated by ultra-relativistic particles, the mean energy per ultra-relativistic particle can be determined from the energy density  $\rho \propto 1/r^2$  and the number-density given in equation (45).

$$\overline{E}(r) = \rho(r)/n(r) \propto \frac{1}{\sqrt{r}} \quad (48)$$

This relation could have been obtained directly from the pressure-induced energy-change of a geodesically moving shell of zero-rest mass particles: As the number of particles in the shell remains constant, but the shell's total energy changes according to  $\sqrt{r_i/r}$ , the mean energy per particle must change in the same way as the total energy of the shell varies.

In a gas of relativistic particles in thermal equilibrium the mean energy per relativistic particle is proportional to the local temperature in appropriate units. This hints at a local radiation temperature within the holostar proportional to  $1/\sqrt{r}$ . This argument in itself is not yet too convincing. It - so far - only applies to the low-density regime in the outer regions of a holostar, where the motion is nearly geodesic and thus interaction free. It is questionable, if a temperature in the thermodynamic sense can be defined under such circumstances.

However, there is another argument for a well defined local radiation temperature with  $T \propto 1/\sqrt{r}$ : At the high pressures and densities within the central region of the holostar most of the known particles of the Standard Model will be ultra-relativistic and their mutual interactions are strong enough to maintain a thermal spectrum. The energy-density of radiation in thermal equilibrium is proportional to  $T^4$ . The energy density  $\rho$  of the holostar is known to be proportional to  $1/r^2$ . Radiation will be the dominant contribution to the mass-energy at high temperature, so this argument also hints at a local temperature within the holostar's central region proportional to  $1/\sqrt{r}$ .

Therefore it is reasonable to assume that the holostar has a well defined internal local temperature of its zero-rest mass constituent particles everywhere, i.e. not only in the hot central region, and that this temperature follows an inverse square-root law in  $r$ .

This temperature is isotropic. This statement should be self-evident for the high temperature central region of the holostar, where the radiation has a very short path-length and the interaction time-scale is short. But one also finds an isotropic temperature in the outer regions of a holostar, where the radiation moves essentially unscattered. Because of spherical symmetry, only radiation arriving with a radial component of the motion at the detector need be considered. Imagine a photon emitted from the hot inner region of the holostar

with an energy equal to the local temperature at the place of emission,  $r_e$ . Due to the square-root dependence of the temperature, the local temperature at the place of emission,  $r_e$ , will be higher than the local temperature at the place of the detector,  $r_a$ , by the square-root of the ratio  $r_a/r_e$ . However, on its way to the detector the photon will be red-shifted due to the pressure-effect by exactly the same (or rather inverse) square-root factor, so that its energy, when it finally arrives at the detector, turns out to be equal to the local temperature at the detector. The same argument applies to a photon emitted from the low-temperature outer region of the holostar. Due to the pressure effect this photon will be blue-shifted when it travels towards the detector. Generally one finds, that the pressure induced red-shift (or blue-shift) exactly compensates the difference of the local temperatures between the place of emission,  $r_e$ , and the place of absorption  $r_a$  of a zero rest mass particle.

Disregarding pressure effects, one could naively assume that an individual photon emitted from an interior position  $r_i$  would undergo gravitational blue shift, as it moves "down" in the effective potential towards larger values of  $r$ . If this were true, a photon moving from a "hot" inner position to a "cold" outer position would become even hotter. This result is paradoxical. In fact, the apparent energy change due to the naive application of the gravitational redshift-formula is exactly opposite to the pressure-induced effect:

$$\frac{\nu}{\nu_i} = \sqrt{\frac{g_{00}(r_i)}{g_{00}(r)}} = \sqrt{\frac{r}{r_i}}$$

Therefore the naive application of the gravitational Doppler-shift formula to the interior space-time of the holostar leads to grave inconsistencies. In the interior of the holostar the usual gravitational Doppler-shift formula is not applicable.<sup>17</sup> This has to do with the fact, that its derivation requires not only a stationary space-time, but also relies on the geodesic equations of motion, which are only the "true" equations of motion in vacuum. Although particles move on geodesics in the (rather unrealistic) case of a pressure-free "dust-universe", this is not true when significant pressures are present.<sup>18</sup> Therefore one should not expect the gravitational Doppler-shift law to be applicable in space-time regions where the pressure is significant.

Note finally, that the frequency shift due to the interior pressure applies to all zero rest-mass particles. Furthermore, the pressure-induced frequency shift is insensitive to the route travelled by the zero rest-mass particles. It solely

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<sup>17</sup>Meaning, that the gravitational Doppler shift is not the only effect that influences the frequency of a photon as a function of position.

<sup>18</sup>This fact can be experienced by anyone living on the surface of the earth. None of us, except astronauts in space, move geodesically. Geodesic motion means free fall towards the earth's center. The pressure forces of the earth's surface prevent us from moving on such a trajectory. From the viewpoint of general relativity the earth's surface exerts a constant force accelerating any object lying on its surface against the direction of the "gravitational pull" of the earth.

depends on how the volume available to an individual particle has changed, i.e. only depends on the number-density of the particles which is a pure function of radial position. If  $r_a$  is the radial position where the photon that was emitted from  $r_e$  with frequency  $\nu_e$  is finally absorbed, one finds.

$$\frac{\nu_e}{\nu_a} = \sqrt{\frac{r_a}{r_e}} \quad (49)$$

It is now an easy exercise to show, that any geodesically moving shell of zero rest mass particles preserves the Planck-distribution:

The Planck-distribution is defined as:

$$n(\nu, T) \propto \frac{\nu^2 d\nu}{e^{\frac{\nu}{T}} - 1}$$

$n$  is the density of the photons,  $\nu$  their individual frequency and  $T$  the temperature. The left side of the equation, i.e. the number density of the photons with a given frequency, scales as  $1/r^{3/2}$  according to equation (45). The right side of the equation has the same dependence. The frequency  $\nu$  of any individual photon scales with  $1/r^{1/2}$  according to equation (49). The same shift applies to the frequency interval  $d\nu$ . Therefore the factor  $\nu^2 d\nu$  on the right side of the equation also scales as  $1/r^{3/2}$ . The argument of the exponential function,  $\nu/T$  is constant, because both the frequency of an individual photon  $\nu(r)$ , as well as the overall temperature  $T(r)$  have the same  $r$ -dependence.

We find that the Planck-distribution is preserved by the geodesic motion of a non-interacting gas of radiation within the holostar. This astonishing result directly follows from the holographic metric, the effects of the negative radial pressure and self-consistency.

The local temperature law for the holostar allows a quick derivation of the Hawking temperature: The zero rest-mass particles at the surface of the holostar will have a local temperature proportional to  $1/\sqrt{r_h}$ . If this is the true surface temperature of the holostar, one should be able to relate this temperature to the Hawking temperature of a black hole. The Hawking temperature is measured in the exterior space-time at spatial infinity. As the exterior space-time is pressure free, any particle moving out from the holostar's surface to infinity will undergo "normal" gravitational red-shift. The red-shift factor is given by the square-root of the time-coefficient of the metric at the position of the membrane, i.e.  $\sqrt{r_0/r_h}$ . Multiplying the local temperature with this factor gives the temperature of the holostar at infinity. We find  $T \propto 1/r_h = 1/(r_+ + r_0)$ . Disregarding the rather small value of  $r_0$  the Hawking temperature of a black hole has exactly the same dependence on the gravitational radius  $r_+ = r_h - r_0$  as the holostar's local surface temperature, measured at infinity. We have just derived the Hawking temperature up to a constant factor. A simple dimensional analysis shows

that the factor is of the order unity. In [30] a more definite relationship will be derived.

One can use the Hawking temperature to fix the local temperature at the holostar's membrane. The local temperature of the membrane is blue-shifted with respect to the Hawking temperature, due to the exterior gravitational field of the holostar. Under the assumption that the blue-shifted Hawking temperature,  $T_H$ , is equal to the local temperature of the membrane,  $T(r_h)$ , we find:

$$T(r_h) = T_H * \sqrt{\frac{B(\infty)}{B(r_h)}} = \frac{\hbar}{4\pi\sqrt{r_h r_0}} \quad (50)$$

Knowing the local temperature within the holostar at one point allows one to determine the temperature at an arbitrary internal position:

$$T = \frac{\hbar}{4\pi\sqrt{r r_0}} \quad (51)$$

With the above equation for the local temperature, the unknown length parameter  $r_0$  can be estimated. Raising equation (51) to the fourth power gives:

$$T^4 = \frac{\hbar^4}{2^5 \pi^3 r_0^2} \frac{1}{8\pi r^2} = \frac{\hbar^4}{2^5 \pi^3 r_0^2} \rho \quad (52)$$

which implies:

$$\frac{r_0^2}{\hbar} = \frac{\hbar^3}{2^5 \pi^3} \frac{\rho}{T^4} \quad (53)$$

Under the assumption, that we live in a large holostar of cosmic proportions we can plug in the temperature of the cosmic microwave background radiation (CMBR) and the mean matter-density of the universe into the above equation. If the recent results from WMAP [37] are used, i.e.  $T_{CMBR} = 2.725 K$  and  $\rho \approx 0.26 \rho_{crit}$ , where  $\rho_{crit} = 3H^2/(8\pi)$  is determined from the Hubble-constant which is estimated to be approximately  $H \approx 71(km/s)/Mpc$ , we find:

$$r_0^2 \approx 3.52\hbar \quad (54)$$

which corresponds to  $r_0 \approx 1.88\sqrt{\hbar}$ . Therefore  $r_0$  is roughly twice the Planck-length, which is quite in agreement to the theoretical prediction  $r_0 \approx 1.87 r_{Pl}$  at low energies, obtained in [28].

## 2.11 Geodesic acceleration and pressure - necessary conditions for nearly geodesic motion of massive particles

In this and the following sections the radial motion of non zero rest-mass (massive) particles will be analyzed in somewhat greater detail. The main purpose of this analysis is to show, that as in the case of photons, geodesic motion of massive particles is self-consistent within the holostar solution, i.e. preserves the energy-density  $\rho \propto 1/r^2$ . It cannot be stated clearly enough, though, that for massive particles geodesic motion is at best an approximation to the true motion of the particles within the pressurized fluid consisting of massive particles and photons alike. The radial pressure of the space-time will always exert an acceleration on a massive particle, so that massive particles can never move truly geodesically. Furthermore, the higher relativistic the motion of a massive particle becomes, the higher its proper acceleration in its own frame will be, because the proper acceleration grows with  $\gamma^3$ . Therefore the results derived in this and the following sections might only be interpretable as "thought-experiments", giving an answer to the question of what would happen, if the motion were geodesic in full. Such a thought-experiment is nonetheless worthwhile, because the "true" motion of the massive particles will lie somewhere "in between" the motion of photons and the geodesic motion of the massive particles. If both, the motion of photons and the geodesic motion of massive particles can be shown to be compatible with the holostar's internal energy density, it is quite likely that any other motion, geodesic or not, will be compatible as well.

The motion of a massive particle in the holostar is subject to two effects: Geodesic proper acceleration and acceleration due to the pressure forces.

The geodesic (proper) acceleration  $g$  for a massive particle at its turning point of the motion, i.e. at the position where it is momentarily at rest with respect to the  $(t, r, \theta, \varphi)$  coordinate system, is given by the following expression:

$$g = \frac{d\beta_r}{d\tau} = \frac{1}{2} \sqrt{\frac{r_0}{r}} \frac{1}{r} \quad (55)$$

In the interior of the holostar, the geodesic acceleration is always radially outward directed, whereas it is inward-directed in the exterior space-time. We find that the geodesic acceleration is always directed towards the membrane. In a certain sense the membrane can be considered as the true source of the gravitational attraction.<sup>19</sup>

Due to the negative radial pressure, an interior (massive) particle will also be subject to a radially inward directed proper acceleration resulting from the "pressure force".

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<sup>19</sup>Note, that the sum  $\rho + P_r + 2P_\theta$ , which when integrated over the whole space-time gives the so called Tolman-mass (sometimes also referred to as the "active gravitational mass"), is zero everywhere except at the membrane.

Under the rather bold assumption, that the negative radial pressure in the holostar is produced in the conventional sense, i.e. by some yet to be found "pressure-particles" moving radially inward, which once in a while collide with the massive particles moving outward, the momentum transfer in the collision process will result in a "net-force" acting on the massive particles. The acceleration by the pressure force,  $a_P$ , can then be estimated as follows for a particle momentarily at rest in the  $(t, r, \theta, \varphi)$ -coordinate system:

$$\frac{dp}{d\tau} = ma_P = P_r \sigma = -\frac{\sigma}{8\pi r^2} \quad (56)$$

$m$  is the (special relativistic) mass-energy of the particle and  $\sigma$  its cross-sectional area with respect to the "pressure-particles". The cross-sectional area will depend on the characteristics of the field (or particles) that generates the radial pressure.  $\sigma$  might also depend on the typical interaction energy at a particular  $r$ -value. If the pressure is gravitational in origin, one would expect the cross-sectional area to be roughly equal to the Planck-area.

The geodesic acceleration  $g$  has a  $1/r^{3/2}$ -dependence, whereas the pressure-induced acceleration  $a_P$  follows an inverse square law ( $a_P \propto 1/r^2$ ), whenever  $\sigma$  and  $m$  can be considered constant. For ordinary matter and the strong and electro-weak forces this condition is fulfilled whenever the particles are non-relativistic<sup>20</sup>, which will be true in the outer regions of the holostar. For large  $r$  it should be possible to neglect the pressure-induced acceleration with respect to the geodesic acceleration. In this case the motion of a massive particle will become geodesic to a high degree of precision. The region of "almost" geodesic motion is characterized by  $g \gg a_P$ , which leads to:

$$r \gg \left( \frac{\sigma}{4\pi} \frac{1}{m} \right)^2 \frac{1}{r_0} \quad (57)$$

If cross-sectional areas typical for the strong force ( $\sigma_S \approx 36\pi\hbar^2/m_p^2 \approx 40mb$ ) are used, the radial coordinate position  $r_{eq}$  where geodesic and pressure-induced acceleration become equal is given by:

$$r_{eq} \approx \left( \frac{9\hbar^2}{m_p^3} \right)^2 \frac{1}{r_0} \approx 10^{83} cm \quad (58)$$

This is roughly a factor of  $10^{55}$  larger than the radius of the observable universe.

For a cross-sectional area of roughly the Planck-area ( $\sigma \approx r_0^2$ ) and for a particle with the mass of a nucleon ( $m \approx 10^{-19}r_0$ ) we find  $r_{eq} \approx 10^{36}r_0 \approx 17m$ ,

<sup>20</sup>Note that the way equation (56) is set up,  $m$  is not the rest-mass of the particle, but rather the "relativistic mass". For relativistic particles  $m$  has to be replaced by  $m \rightarrow E = m\gamma$ .

roughly 0.5 percent of the gravitational radius of the sun. According to equation (51) the local temperature of the holostar at this point is roughly  $T_{eq} \approx 10^{13}K$ . This corresponds to a thermal energy of roughly  $1GeV$ , i.e. the rest mass of the nucleon. For an electron the radial position of equal geodesic and pressure induced acceleration will be larger by the squared ratio of the proton-mass to the electron mass:  $r_{eq} \approx 5.8 \cdot 10^4 km$ . At this point the local temperature is roughly  $T_{eq} \approx 5 \cdot 10^9 K$ , corresponding to an energy of  $500keV$ , i.e. the rest mass of the electron.

Using equation (51) the inequality in equation (57) can be expressed as:

$$m > \frac{\sigma}{4\pi\sqrt{r_{eq}r_0}} = \frac{\sigma}{\hbar}T \quad (59)$$

In the following discussion I make the assumption, that the cross-sectional area  $\sigma$  of the pressure-particles is comparable to the Planck-area, i.e.  $\sigma \approx A_{Pl} \approx \hbar$ . From equation (59) we find, that whenever the local temperature of one of the constituent particles of the holostar falls below its rest-mass, the outward directed geodesic acceleration becomes larger than the inward directed acceleration due to the pressure, allowing the particle to move outward on a trajectory which becomes more and more geodesical.

What happens in the region of the holostar, where the temperature is higher than the rest-mass of the particle? In this case, the mass  $m$  in equation (59) must be replaced by the total mass-energy of the particle,  $m \rightarrow E = \sqrt{m^2 + p^2}$ . Equation (59), which is the condition of zero net acceleration, then reflects the trivial condition  $E \simeq T$ . This condition is fulfilled, at least approximately, whenever the local temperature is higher than the rest mass of the (massive) particles. Therefore we arrive at the remarkable conclusion, that whenever the interior particles become ultra-relativistic, their net-acceleration in the frame of the observer at rest in the  $(t, r, \theta, \varphi)$  coordinate system is nearly zero, i.e. the holostar is nearly static in this regime.

In [30] it is shown, that  $E = sT$  for ultra-relativistic particles within the holostar.  $s$  is the entropy per relativistic particle, which is nearly constant and lies in the range between 3.15 and 3.5 for reasonable assumptions concerning the ratio of bosonic to fermionic degrees of freedom.<sup>21</sup> The condition for zero net-acceleration in the radiation dominated central region of the holostar will be fulfilled exactly, if

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<sup>21</sup> $s = 3.37$  is the preferred value for the entropy per particle. As shown in [30] the mean entropy per particle,  $s$ , attains this value for the supersymmetric high temperature phase, where all particles are ultra-relativistic, the fermionic and bosonic degrees of freedom are equal and the chemical potentials of fermions and bosons are opposite to each other. Keep in mind, that  $s$  is the mean entropy per particle. In the holostar bosons and fermions have different entropy. Therefore the relation  $E = sT$  doesn't apply to bosons or fermions individually, but rather to the average of both species. In order to keep things simple, I will not make this necessary distinction here.

$$\frac{E}{T} = \frac{\sigma}{\hbar} = s \quad (60)$$

The temperature  $T_i$ , where a particle with rest mass  $m$  starts to move outward, is given by:

$$T_i = \frac{m}{s} \approx \frac{m}{3} \quad (61)$$

It is not altogether clear, whether zero net-acceleration is exactly achievable in the hot interior region of the holostar. For  $\sigma/\hbar < s$  the geodesic outward directed acceleration is larger than the pressure-induced deceleration. If the holostar solution is combined with the results of quantum gravity and the Immirzi parameter is fixed at  $\gamma = s/(\pi\sqrt{3})$  (see the discussion in [28]) then  $\sigma/\hbar = s$ , at least at the Planck energy. At lower energies the mean cross-sectional area of the particles is expected to lie in the range  $\pi\sqrt{3}/4 < \sigma/\hbar < s$  (for a fundamental spin-1/2 particle). Therefore for spin-1/2 fermions we expect the mean cross-sectional area  $\sigma$  (with respect to the pressure forces) to be always less than the mean entropy per particle  $s$ , except at the central region  $r \approx r_0$ .

## 2.12 A possible origin of the negative radial pressure

Note that this section is highly speculative and trods into uncharted territory without the appropriate guide. On the other hand, the issues addressed here must be solved in one way or the other, if the holostar is to be a truly self-consistent model, not only for black holes comparable to the mass of the sun, but for holostars of arbitrary size, up to and exceeding the observable radius of the universe. It might well be that the solution to the problems addressed in this section, if there is any, will turn out to be completely different from what is presented here.

It is an astonishing coincidence, that the local temperature of the holostar at the radial position  $r_{eq}$ , i.e. where the net acceleration (geodesic and pressure-induced) becomes nearly zero for a particular particle, is roughly equal to the rest mass of the particle. At least this is the case, if the result  $\sigma \approx s\hbar$ , which was obtained independently in [28], is used. This can be considered as indication, that the negative radial pressure within the holostar is not just a curious mathematical feature of the solution, but could be a real, measurable physical effect, and that the assumption that the pressure is gravitational in origin, with a cross-sectional area  $\sigma$  comparable to the Planck-area, is not too far off the track.

In fact, if the pressure were produced by a continuous flow of particles moving radially inward, and that interact only very weakly with the outflowing "ordinary matter", this could explain the purely radial nature of the pressure. If the inflowing "pressure-particles" carry a mass-energy equivalent to that of

the outflowing ordinary matter, this could at the same time explain the mystery, that the holostar is a static solution which requires that any outflow of mass-energy must be accompanied by an equivalent inflow.<sup>22</sup> Therefore we arrive at two conditions that should be met by the "pressure particles", whenever "ordinary matter" and "pressure-particles" must be regarded as distinct:

- The pressure particles should interact very weakly with ordinary matter (at least at energies below roughly 1 MeV)
- the mass-energy density of the pressure particles should be equivalent to the mass-energy density of ordinary matter

The first condition can - in principle - be fulfilled by several particles. The graviton, the supposed messenger particle for the gravitational force, looks like the most suitable candidate. The second condition suggests that supersymmetric matter might be the preferred candidate for the pressure-particles, if they exist: It seems quite improbable that massless gravitons, with only two degrees of freedom, can deliver an energy flow exactly equal to the energy flow of ordinary matter at any arbitrary radial position within the holostar. But exact supersymmetry predicts equal numbers<sup>23</sup> and masses of the supersymmetric particles. If the "pressure particles" were the supersymmetric partners

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<sup>22</sup>There is no mystery, as long as the outmoving particles reverse their outward directed motion in the exterior space-time and thus "swing" back and forth between exterior and interior space-time (without friction), as suggested by the holostar equations of motion for a stable massive particle. The motion of the massive particles would be time-symmetric and any outmoving particle would be confronted by a highly relativistic flux of particles moving inward. The mystery arises, when we identify the holostar with the observable universe. It will be shown later, that any outmoving massive particle experiences an isotropic radially outward directed Hubble-flow in its co-moving frame. This is exactly what we ourselves observe. However, our situation in the expanding universe appears time-asymmetric: We haven't yet noticed a flow of particles hitting us head-on in our "outward" directed motion (although we might already have noticed the effects of the inflowing matter in the form of a positive cosmological constant, i.e. a negative pressure). This must not altogether be a contradiction. First, if the outflowing and inflowing matter moves highly relativistically, as is expected from the holostar equations of motion for a single massive particle, the cross-sectional area for a collision between an outmoving "matter particle" and an in-moving "pressure particle" can become almost arbitrarily small, as  $\sigma \propto \hbar^2/E^2$  for ordinary matter. If the collision energy in the center of mass frame (=coordinate frame) is of order of the Planck energy, the cross-sectional area will be comparable to the Planck area. For larger collision energies a black hole will be formed, so that the cross-sectional area is expected to rise as the surface area of a black hole, which is proportional to  $E^2$ . All in all we expect a cross-sectional area of  $\sigma \propto \hbar^2/E^2 + E^2$ . Secondly, the outflowing particles and the inflowing particles must not necessarily be the same. The inflowing particles could be decay products of the outmoving particles or altogether different species. Any weakly interacting "dark matter" particle would probably qualify. If the inflowing particles interact only weakly with the outflowing "ordinary" matter, they could deliver an inward-directed energy flow comparable to the outward directed flow and a time-symmetric situation with respect to the net energy flow could be restored.

<sup>23</sup>With equal numbers I don't mean equal number densities per proper volume, but rather equal numbers of degrees of freedom, i.e. equal number of particle species. The number densities of bosons and fermions in thermodynamic equilibrium in the holostar will be different, even if the number of fermionic and bosonic species are equal (for a detailed discussion see [28]).

of ordinary matter, it might be possible to fulfil the second condition quite trivially.<sup>24</sup> Furthermore, for interaction energies below the electro-weak unification scale supersymmetric matter and ordinary matter are effectively decoupled from each other. Supersymmetric matter therefore fits both conditions well. Note also, that the holostar solution assumes that the cosmological constant is exactly zero. Exact supersymmetry guarantees just this. Therefore - from a purely theoretical point of view - we would get a much more consistent description of the phenomena, if supersymmetry were realized exactly even in the low density / low energy regions of a large holostar.

If the principal agents of the negative radial pressure consist of supersymmetric matter, this requires an efficient mechanism which converts the outflowing ordinary matter into the supersymmetric "pressure-particles". This process is expected to take place in the membrane. The membrane is in many respects similar to a two-dimensional domain wall. Therefore the membrane could induce interactions similar to the conversion processes that are believed to take place when ordinary matter crosses a two-dimensional domain wall. On the other hand, if the lightest supersymmetric particle were light enough, ordinary matter could just decay into the lightest superparticle anywhere on its way outward.<sup>25</sup>

Supersymmetry might also provide the solution to the problem, why the "pressure particles" (bosons) preferably move inward, whereas ordinary matter (fermions) moves outward.<sup>26</sup> Such behavior would be easier to understand if one could think up a mechanism that explains this time-asymmetric situation: If geodesic movement in the holostar were fully  $T$ -symmetric, the time reversed process would be equally likely, in which case the "pressure-particles" should be ordinary matter.<sup>27</sup> In the following argument I make use of the fact, that the membrane appears as the primary source of gravitational attraction in the low density region of the holostar, at least for ordinary matter in nearly geodesic motion. Could the membrane expel the "pressure particles", after the interactions in the membrane have converted ordinary matter into supersymmetric "pressure particles"? The gravitational force is always attractive, unless we were able to find some form of matter with  $M^2 < 0$ .

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<sup>24</sup>If supersymmetry is broken, the masses of the superparticles cannot be much higher than the  $W$ - or  $Z$ -mass. In this case the negative radial pressure could still be generated by an appreciable, but not too high number-density of the lightest superparticle (LSP).

<sup>25</sup>The time of decay would have to be roughly equal to the radius of the holostar. As the proton's lifetime is several orders of magnitude larger than the current age of the universe, this would require a rather large holostar/universe.

<sup>26</sup>As has been shown in the previous section, the problem only exists in the low-density (dynamic) regions of the holostar, where massive particles move geodesically. The high temperature central region of the holostar poses no such problem: Due to the approximate balance between pressure-induced and geodesic acceleration, the holostar is expected to be nearly static and time-symmetric in this region.

<sup>27</sup>In fact, if the "pressure-particles" move fast enough, they could well be ordinary matter (see footnote 22). An interaction energy between outflowing ordinary matter and inflowing pressure particles of order of the Planck energy doesn't seem too far fetched, if the discussion in sections 2.18 and 2.19 is taken seriously.

This is the point where supersymmetry might come up with an answer: The Higgs-field, which is expected to give mass to the particles of the Standard Model, is characterized by a quantity  $M^2$ , which is usually identified with the mass squared of the field. Whenever  $M^2 > 0$ , all particles of the Standard model are massless. Whenever  $M^2$  falls below zero, the Higgs-mechanism kicks in. In the supersymmetric extension of the Standard Model the dependence of  $M^2$  on the energy/distance-scale can be calculated. It turns out that  $M^2$  is positive at the Planck-scale and becomes negative close to the energy scale of the Standard-Model. Therefore the condition  $M^2 < 0$  will be fulfilled in the problematic low-density region of the holostar, i.e. whenever  $T < M_{Higgs}$ . The peculiar property  $M^2 < 0$  of the Higgs-field at low energies therefore might provide the mechanism, by which supersymmetric matter is rather expelled from the membrane, whereas ordinary matter is attracted. If supersymmetry can actually provide such a mechanism, we could understand why the holostar is a static solution, not only for small holostars<sup>28</sup>, but also for an arbitrarily large holostar, where a large fraction of the interior matter is situated in the regime where the Higgs-field(s) have  $M^2 < 0$  and "ordinary matter" moves outward.

### 2.13 Nearly geodesic motion of massive particles

In the following we are interested in the low density regions of the holostar, where the motion of massive particles should become more and more geodesic, i.e. for  $T \ll m$ . The geodesic equation of pure radial motion for a massive particle, starting out at rest from  $r = r_i$ , is given by equation (25):

$$\beta_r = \frac{dr}{dt} \frac{r}{r_0} = \sqrt{1 - \frac{r_i}{r}} \quad (62)$$

Integration of the above equation gives:

$$2r_0 t = \sqrt{1 - \frac{r_i}{r}} (r^2 + \frac{3r_i}{2}r) + \frac{3}{4}r_i^2 \ln \left( \frac{2r}{r_i} (\sqrt{1 - \frac{r_i}{r}} + 1) - 1 \right) \quad (63)$$

For  $r \gg r_i$  the logarithm can be neglected and the square-root can be Taylor-expanded to first order:

$$2r_0 t \approx r^2 + r_i r \quad (64)$$

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<sup>28</sup>For small holostars, where  $M_H^2 > 0$  in the whole interior, all of its constituent particles should be massless. For massless particles the condition  $T \approx E$  is trivially fulfilled. If  $\sigma/\hbar = s = E/T$  the pressure-induced inward directed acceleration and the geodesic outward directed acceleration are equal throughout the whole interior. In such a case the holostar is truly static, time-symmetric and in thermal equilibrium. Note also, that an extended static central region provides an excellent "protection" against continued gravitational contraction of the whole space-time to a point-singularity.

or

$$r \approx -\frac{r_i}{2} + \frac{1}{2}\sqrt{r_i^2 + 8r_0 t} \quad (65)$$

Two massive particles separated initially at  $r_i$  by a radial coordinate separation  $\delta r_i$  and moving geodesically outward, will have a radial coordinate separation  $\delta r(r)$  that tends to the following constant value when  $r \gg r_i$

$$\delta r(r) \rightarrow -\frac{\delta r_i}{2} \quad (66)$$

The minus sign in the above equation is due to a "cross-over" of the massive particles, which takes place at a very early stage of the motion, i.e. where  $r \approx r_i$ . After the cross-over the radial coordinate separation of the massive particles quickly approaches the value  $\delta r_i/2$  and remains effectively constant henceforth.

Constant coordinate separation means that the proper radial separation viewed by an observer at rest in the  $(t, r, \theta, \varphi)$ -coordinate system,  $\delta l$ , develops according to:

$$\delta l(r) = \delta l_i \sqrt{\frac{r}{r_i}} \quad (67)$$

Therefore, quite contrary to the movement of the zero-rest mass particles, an outmoving shell of massive particles expands along the radial direction.

Due to the negative radial pressure the energy in the shell increases as the shell expands radially. A similar calculation as in the zero rest mass case gives the following total energy change in the shell:

$$\delta E(r) = \delta E_i \sqrt{\frac{r}{r_i}} \quad (68)$$

The proper volume of the shell, as measured from an observer at rest in the chosen coordinate system, changes according to:

$$\delta V(r) = \delta V_i \left( \frac{r}{r_i} \right)^{\frac{5}{2}} \quad (69)$$

A factor proportional to  $r^2$  comes from the proper surface area of the shell, a factor of  $r^{1/2}$  from the proper expansion of the shell's radial dimension.

Note that exactly as in the case of zero rest-mass particles the mass-energy density of the expanding shell, viewed from an observer at rest, follows an inverse square law.

$$\rho(r) = \frac{\delta E(r)}{\delta V} = \frac{\delta E_i \sqrt{\frac{r}{r_i}}}{\delta V_i \left(\frac{r}{r_i}\right)^{\frac{5}{2}}} = \rho_i \left(\frac{r_i}{r}\right)^2 \quad (70)$$

Therefore the geodesic motion of a shell of massive particles also is self-consistent with the static holographic mass-energy density. In fact, this self-consistency is independent of the path of the motion and the nature of the particle. It only depends on the radial pressure which guarantees, that for any conceivable motion, geodesic or not, the energy of the particles will change such that the energy-density law  $\rho \propto 1/r^2$  within the holostar is preserved.

Under the assumption that no particles are created or destroyed in the shell, the number-density of massive particles develops according to the inverse proper volume of the shell, as no massive particles can leave the shell in the regime, where  $\delta r_i = \text{const.}$

$$n_m(r) = n_i \left(\frac{r_i}{r}\right)^{\frac{5}{2}} \quad (71)$$

In section 2.22 it will be shown that expansion against the negative pressure has the effect to produce new particles in the co-moving frame. Therefore the assumption that the particle-number remains constant in the shell, is not entirely correct. On the other hand, genuine particle-production in the shell must obey the relevant conservation laws. Some of those, such as conservation of lepton- and baryon-number, are empiric. Those "empiric" conservation laws can be violated without severe consequences with respect to our established physical theories. There are only few conservation laws that are linked to first principles, such as local gauge-symmetries. One of these conservation laws is charge-conservation. Therefore particle creation in the shell must observe charge conservation. As charge is quantized in units of the electron charge the difference between positively and negatively charged elementary particles in the shell must remain constant, so that the net number-density of charged particles scales with  $1/r^{5/2}$ .

Even if particle production in the co-moving shell is taken into account, the number-density of the massive particles declines much faster than the number-density of the zero-rest mass particles. This is a consequence of the fact, that a geodesically moving shell of photons is compressed in the radial direction, whereas a geodesically moving shell of massive particles expands. If no particles are created or destroyed, the respective number densities behave differently.

The ratio of zero-rest mass particles to massive particles should be independent of the local Lorentz frame of the observer (see however the discussion in section 2.22). Therefore, whenever the zero-rest mass particles are chemically and kinematically decoupled from the non-zero rest-mass particles, their ratio can be used as a "clock" by an observer co-moving with the massive particles. However, any net momentum transfer between massive and zero rest-mass

particles will distort this relation. Therefore this particular "clock" cannot be considered as highly accurate.

According to equation (68) the energy of the shell increases with  $\sqrt{r/r_i}$ . If we assume no particle creation in the shell, each massive particle must acquire an increasingly larger energy  $E = m_0\sqrt{r/r_i}$ . Where does this energy come from?

The energy is nothing else than the energy of the motion of a massive particle, as viewed by an asymptotic observer at rest with respect to the  $(t, r, \theta, \varphi)$  coordinate system. According to the equations of motion the (almost) radial velocity, measured by an observer at rest in the  $(t, r, \theta, \varphi)$  coordinate system is given by:

$$\beta^2(r) \simeq \beta_r^2(r) = 1 - \frac{r_i}{r} \quad (72)$$

which implies:

$$\gamma^2(r) = \frac{r}{r_i} \quad (73)$$

The total energy  $E(r)$  of a massive particle with rest mass  $m$ , viewed by an observer at rest in the coordinate system, will then be given by:

$$E(r) = \gamma(r)m_0 = m_0\sqrt{\frac{r}{r_i}} \quad (74)$$

This is just the energy-increase per particle, which has been derived from the pressure-induced increase due to the radial expansion of the shell against the negative pressure.

Therefore, from the perspective of an exterior observer, energy is conserved not only locally, but globally in the holostar space-time. This is remarkable, because global energy conservation is not mandatory in general relativity. In fact, there exist only a limited class of space-times (such as asymptotically flat space-times) in which a global concept of energy can be rigorously defined. Except for a small class of symmetric space-times it is generally impossible to define a locally meaningful concept of gravitational energy.<sup>29</sup>

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<sup>29</sup>So far no realistic cosmological space-time has been found, in which global energy-conservation holds. In the standard cosmological models global energy-conservation is heavily violated in the radiation dominated era. If the holostar turns out to be a realistic alternative to the standard cosmological models, and energy is conserved globally in the holostar space-time, one could use global energy conservation as a selection principle to choose among various possible solutions.

## 2.14 Entropy area law

For a large holostar the total number of non-relativistic (massive) particles ( $N_m \propto r_h^{3/2}$ ) can be neglected with respect to the number of ultra-relativistic (zero rest-mass) particles ( $N \propto r_h^2$ ). According to the  $T \propto 1/\sqrt{r}$ -law the main contribution to the mass of a large holostar comes from its outer low temperature regions. But whenever the temperature becomes lower than the rest mass of the particle, the number density of the non-relativistic massive particles is thinned out with respect to the number density of the yet relativistic particles. On the other hand, for a small holostar the internal local temperature is so high, that the majority of massive particles will become ultra-relativistic, in fact massless whenever the Higgs-mechanism fails to function, because  $M_H^2 > 0$ . Therefore the dominant particle species of a holostar, large or small, will be ultra-relativistic or zero rest mass particles.

Under the reasonable assumption that the entropy of the holostar is proportional to the number of its particles, one recovers the Hawking entropy-area law for black holes up to a constant factor.<sup>30</sup> A dimensional analysis shows, that the factor is of order unity. We therefore find:

$$S \propto N \propto \frac{A}{\hbar} \quad (75)$$

A more definite relationship will be derived in [30].

## 2.15 Motion of massive particles in their own proper time

In this section I will examine the equations of motion from the viewpoint of a moving massive particle, i.e. from the viewpoint of the co-moving material observer who moves geodesically. The geodesic equation of radial motion for a massive particle, expressed in terms of its own proper time, is given by:

$$\frac{dr}{d\tau} = \sqrt{\frac{r_0}{r_i}} \sqrt{1 - \frac{r_i}{r}} \quad (76)$$

The radial coordinate velocity  $dr/d\tau$  is nearly constant for  $r \gg r_i$ . Integration of the above equation gives:

$$\tau = \sqrt{\frac{r_i}{r_0}} \left( r \sqrt{1 - \frac{r_i}{r}} + \frac{r_i}{2} \ln \left( \frac{2r}{r_i} \left( \sqrt{1 - \frac{r_i}{r}} + 1 \right) - 1 \right) \right) \quad (77)$$

For large  $r \gg r_i$  this can be simplified:

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<sup>30</sup>with a presumably small correction for the massive particles

$$\tau \cong \sqrt{\frac{r_i}{r_0}} \left( r + \frac{r_i}{2} \ln \frac{4r}{r_i} \right) \cong \sqrt{\frac{r_i}{r_0}} r \quad (78)$$

The proper time it takes a material observer to move along a radial geodesic trajectory through the holostar space-time is proportional to  $r$ . This is very much different from what the external asymptotic observer sees. The time measured by an exterior clock at infinity has been shown to be proportional to  $r^2$ .

Under the assumption that we live in a large holostar formula (78) is quite consistent with the age of the universe, unless  $r_i \ll r_0$ .<sup>31</sup> As can be seen from equation (78), it takes a material observer roughly the current age of the universe in order to travel geodesically from the Planck-density region at the holostar's center (i.e. at  $r_i \approx r_0$ ) to the low density region at  $r \approx 10^{61} r_0$ , where the density is comparable to the density of the universe observed today: For  $r_i = r_0$  we find  $\tau = r \approx 1.6 \cdot 10^{10} y$ , if  $r \approx 10^{61} r_{Pl}$ . The proper time of travel could be much longer: If a massive particle is emitted (with zero velocity) from  $r_i > r_0$ , the proper time of travel to radial position  $r$  is larger than the former value by the square root ratio of  $r_i$  to  $r_0$ . Therefore the holostar solution is compatible with the age of the oldest objects in our universe ( $\approx 1.3 - 1.9 \cdot 10^{10} y$ ), but would also allow a much older age.

Note, that the holostar solution has no need for inflation. According to the equations governing the geodesic motion of a material observer in the holostar, the whole observable universe has started out from a space-time region in thermal equilibrium, which was causally connected. The "scale factor"  $r$  develops proportional to  $\tau$ . The "expansion", defined by the local Hubble-radius, also develops proportional to  $\tau$ . Therefore any causally connected region remains causally connected during the "expansion". The causal horizon and the particle-horizon remain always proportional. Furthermore the number-density law  $n_m \propto 1/r^{5/2}$  for massive particles indicates, that very massive particles that have decoupled kinematically from the radiation at an early epoch, such as magnetic monopoles, become very much thinned out with respect to the radiation or the lighter particles, such as baryons, which decouple much later.

## 2.16 A linear and a quadratic redshift distance relation

From equation (49) a linear redshift-distance relation can be derived, which is in some sense similar to the redshift-distance relations of the standard Robertson Walker models of the universe.

Imagine a concentric shell of material observers moving radially outward through the holostar. Place two observers in galaxies at the inner and outer

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<sup>31</sup>As has been noted before, it is quite unlikely, that any particle can be emitted from  $r_i \ll r_0$ .

surfaces of the shell and another observer in a galaxy midway between the two outer observers. When the observer in the middle reaches radial coordinate position  $r_e$ , the two other observers are instructed to emit a photon with frequency  $\nu_e$  in direction of the middle observer.<sup>32</sup> At this moment the proper radial thickness of the shell, i.e. the proper distance between the two outer galaxies, shall be  $\delta l_e$ . Let the three galaxies travel geodesically outward. Some million years later (depending on how large  $\delta l_e$  has been chosen), the photons from the edge-galaxies will finally reach the observer in the middle galaxy. The observer in the middle determines his radial coordinate position at the time of absorption,  $r_a$ . According to equation (49) the photons will have been red-shifted by the squareroot of the ratio of  $r_e/r_a$ . In order to derive the redshift-distance relation we only need to know, how the proper distance between the galaxies has changed as a function of  $r$ . According to equation (67) the proper radial distance grows proportional to the square-root of the radial coordinate value, whenever the galaxies move geodesically:

$$1 + z = \frac{\nu_e}{\nu_a} = \sqrt{\frac{r_a}{r_e}} = \frac{\delta l_a}{\delta l_e} \quad (79)$$

The final result is, that the light emitted from the distant galaxies is red-shifted by the ratio of the proper distances of the galaxies at the time of emission to the time of absorption.

However, this is the result that an observer at rest in the  $(t, r, \theta, \varphi)$ -coordinate system would see. The co-moving observer will find a different relation (see also the discussion in the following section and in section 2.22). For the co-moving observer the proper radial distance has to be multiplied with his special relativistic  $\gamma$ -factor. If we denote the proper separation between the galaxies in the system of the co-moving observer with an overline, we find:

$$\frac{\overline{\delta l_a}}{\delta l_e} = \frac{r_a}{r_e} \quad (80)$$

If we insert this into equation (79) the result is:

$$(1 + z)^2 = \left( \frac{\nu_e}{\nu_a} \right)^2 = \frac{\overline{\delta l_a}}{\delta l_e} \quad (81)$$

This result might seem paradoxical. However, this is exactly what the co-moving observer must see: The stress-energy tensor of the holostar space-time is radially boost invariant. Therefore any radial boost should not affect the local physics. This means that in the co-moving frame, as well as in the coordinate frame, the frequency of the photons should be proportional to the local radiation

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<sup>32</sup>All observers could (at least in principle) synchronize their clocks via the microwave background radiation or the total matter density.

temperature, i.e.  $\nu \propto T$ . The local radiation temperature, however, depends on the inverse square-root of the radial coordinate value:  $T \propto 1/\sqrt{r}$ . Due to Lorentz contraction (or rather Lorentz-elongation in the co-moving frame) the proper distance  $\delta\bar{l}$  in the radial direction between two geodesically moving massive particles develops proportional to  $r$  (see also the next section). Putting all this together gives:  $\delta\bar{l} \propto r \propto 1/T^2 \propto 1/\nu^2 \propto 1/(1+z)^2$ .

The quadratic redshift dependence should show up in the measurement of the Hubble-constant at high redshifts. Its effect should be similar to that of a cosmological constant.

## 2.17 An isotropic Hubble flow of massive particles

With the equations of motion for massive particles one can show, that an observer co-moving with the massive particles within the outmoving shell will see an isotropic Hubble-type expansion of the massive particles with respect to his point of view.

First let us calculate the Hubble-flow viewed by the co-moving observer in the tangential direction, by analyzing how the proper distance between the radially moving observer and a neighboring radially moving particle develops. I.e. observer and particle always have the same  $r$ -coordinate value. Because of spherical symmetry the coordinate system can be chosen such, that both move in the plane  $\theta = \pi/2$ . The observer moves along the radial trajectory  $\varphi = 0$  and the neighboring particle along  $\varphi = \varphi_0$ . The proper distance between observer and particle is given by:

$$l = r\varphi_0$$

After a time  $d\tau$  the distance will have changed due to the radial motion of both particles:

$$\frac{dl}{d\tau} = \frac{dr}{d\tau}\varphi_0$$

The "speed" by which the particle at  $\varphi_0$  moves away from the observer is given by:

$$v = \frac{dl}{d\tau} = \frac{dr}{d\tau}\varphi_0 = \frac{dr}{d\tau} \frac{l}{r}$$

Therefore the Hubble-parameter in the tangential direction is given by:

$$H_{\perp} = \frac{v}{l} = \frac{dr}{d\tau} \cdot \frac{1}{r}$$

Note that special relativistic effects due to the highly relativistic motion of the co-moving observer don't have to be taken into account, because the distances and velocities measured are perpendicular to the direction of the motion.

For the derivation of the Hubble parameter in the radial direction the Lorentz-contraction due to the relativistic motion has to be taken into account. The proper radial separation of two particles in geodesic motion, as seen by the observer at rest in the  $(t, r, \theta, \varphi)$  coordinate system, develops as:

$$l = l_i \sqrt{\frac{r}{r_i}}$$

This formula has to be corrected. Due to the relative motion of the co-moving observer, the observer at rest in the  $(t, r, \theta, \varphi)$  coordinate system will see a proper length, which has been Lorentz-contracted. Therefore the co-moving observer must measure a proper length which is larger by the special relativistic  $\gamma$ -factor. In the system of the co-moving observer the formula for the proper length (denoted by barred quantities) is then given by:

$$\bar{l} = \bar{l}_i \frac{r}{r_i}$$

This gives:

$$H_r = \frac{\frac{d\bar{l}}{d\tau}}{\bar{l}} = \frac{dr}{d\tau} \cdot \frac{1}{r} = H_{\perp} \quad (82)$$

The radial and the tangential local Hubble-values are equal. They just depend on  $r$  and the "proper radial coordinate velocity"  $dr/d\tau$ . Its value is given by equation (76). It is nearly constant for  $r \gg r_i$ . Therefore the isotropic Hubble-parameter can finally be expressed as:

$$H(r) = \frac{1}{r} \sqrt{\frac{r_0}{r_i} - \frac{r_0}{r}} \simeq \frac{1}{r} \sqrt{\frac{r_0}{r_i}} \simeq \frac{1}{\tau} \quad (83)$$

## 2.18 Density perturbations and their evolution in the holostar universe

The Hubble law given in equation (83) contains an unknown parameter, the nearly constant proper radial coordinate velocity  $dr/d\tau \approx \sqrt{r_0/r_i}$ . If the holostar is an appropriate model for the universe,  $dr/d\tau$  can be determined by comparing the measured Hubble constant with our current radial coordinate position  $r$ , which can be estimated from the mass-density. This will be done in section 2.20. However, it would be quite helpful if  $dr/d\tau$  could be determined

by some independent method. The evolution of the density-fluctuations from the time of decoupling to the structures we find today might provide such a means:

The fluctuations in the microwave background radiation have been determined to be roughly equal to  $\delta T/T = 10^{-5}$ . These small temperature fluctuations are interpreted as the relative density fluctuations at the time of decoupling, i.e. at the time when the microwave background radiation temperature was believed to be roughly 1000 times higher than today. In the adiabatic approach (instantaneous decoupling) the respective fluctuations in the baryon-density at the time of decoupling are  $\delta = \delta\rho_B/\rho_B \approx 3\delta T/T$ .<sup>33</sup> From these fluctuations the large scale distribution of galaxies, as we see them today, should have formed. The density fluctuation in the distributions of galaxies on a large scale is roughly of order unity today:  $\delta_{today} = \delta\rho/\rho \simeq 1$ . In the standard cosmological models this evolution of the density fluctuations is quite difficult to explain. The problem is, that in the dust approximation (no significant pressure, velocity of sound nearly zero), which is assumed to be an accurate description of the universe after decoupling, the fluctuations grow with  $\delta \propto t^{2/3}$ . After decoupling the universe is matter dominated. In the matter dominated era  $r \propto t^{2/3}$ . The temperature  $T$  and the scale factor  $r$  are related in the standard cosmological models as  $T \propto 1/r$ . Combining these dependencies we get:

$$\delta \propto t^{2/3} \propto r \propto \frac{1}{T} \quad (84)$$

The above formula predicts  $\delta_{today} \approx 10^{-2}$ , which is roughly two orders of magnitude less than the observed value.

In order to explain the rather large density fluctuations today, the standard cosmological model is usually extended to encompass cold dark matter.

In the holostar universe the evolution of the density-fluctuations can be explained quite easily. Only one parameter,  $dr/d\tau$ , which appears in equation (83) for the Hubble value need to be adjusted.

Let us first consider the dust case, i.e. the evolution of the density fluctuations in the holostar by deliberately ignoring the effects of the pressure. After decoupling the expansion in the holostar universe (in the co-moving frame) is very similar to the expansion in the standard Robertson Walker models. The expansion is isotropic and the density is matter-dominated. Therefore the usual standard model formula for the evolution of the density fluctuations should be applicable. In a matter-dominated Robertson Walker universe with no pressure, the density fluctuations evolve according to the following differential equation (see for example [23]):

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<sup>33</sup>This estimate is based on the assumption, that decoupling happened very fast, almost instantly. Today's refined estimates take into account, that the decoupling took longer. In this case the ratio  $\delta\rho_B/\rho_B$  at decoupling is lower than in the simple adiabatic model up to a factor of 10 (see for example [35]).

$$\ddot{\delta} + 2\frac{\dot{r}}{r}\dot{\delta} - 4\pi\rho\delta = 0 \quad (85)$$

For the holostar we find:

$$r = \sqrt{\frac{r_0}{r_i}}\tau \quad (86)$$

$$\frac{\dot{r}}{r} = \frac{1}{\tau} \quad (87)$$

$$4\pi\rho = \frac{1}{2r^2} = \frac{r_i}{2r_0}\frac{1}{\tau^2} \quad (88)$$

With these relations equation (85) reduces to the following differential equation:

$$\ddot{\delta} + \frac{2}{\tau}\dot{\delta} - \frac{r_i}{2r_0}\frac{1}{\tau^2}\delta = 0 \quad (89)$$

The equation can be solved by the following ansatz:

$$\delta = \tau^n \quad (90)$$

This gives a quadratic equation for  $n$

$$n^2 + n - \frac{r_i}{2r_0} = 0 \quad (91)$$

which can be solved for  $n$ :

$$n = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{r_i}{2r_0}} \quad (92)$$

In the holographic universe we have the following dependencies:

$$r \propto \tau \propto \frac{1}{T^2}$$

Therefore we can express  $\delta$  as a function of temperature  $T$ :

$$\delta \propto \tau^n \propto \frac{1}{T^{2n}} \propto \frac{1}{T^\epsilon} \quad (93)$$

with

$$\epsilon = -1 \pm \sqrt{1 + \frac{2r_i}{r_0}} \quad (94)$$

The exponent  $\epsilon$  in the above equation can be estimated from the known ratio of the density contrast  $\delta_{dec}/\delta_{today}$  and the ratio of the decoupling temperature to the CMBR-temperature.

In the holostar the temperature at decoupling will be larger than in the standard cosmological model, because the number ratio of baryons to photons doesn't remain constant. If radiation and matter do not interact after decoupling (no re-ionization!) and the baryon number remains constant in the co-moving volume, the baryon-density will scale as  $1/r^3 \propto T^6$  in the frame of the geodesically moving material observer. However, the discussion in section 2.22 indicates, that the expansion against the negative pressure creates new particles. The simplest assumption compatible with the experimental data is that the baryon-density will scale as  $1/r^2 \propto T^4$ . Under these circumstances the Saha-equation yields a temperature at decoupling of roughly  $T_{dec} \approx 4900K$ , when no dark matter component is assumed and the baryon-density today is set to the total matter-density as determined by WMAP [37], i.e.  $\rho_B = \rho_m \approx 2.5 \cdot 10^{-27} kg/m^3$ .<sup>34</sup> A decoupling temperature of  $4900K$  corresponds to a red-shift  $z_{dec} \simeq 1790$ . In order to achieve  $\delta_{today} \approx 1$  from  $\delta_{dec} \approx 3 \cdot 10^{-5}$  at  $z_{dec} = 1790$ , the exponent  $\epsilon$  in equation (93) must be roughly equal to 1.48. For more realistic scenarios in which decoupling doesn't happen instantaneously  $\delta_{dec}$  is estimated to be lower, up to a factor of 10 [35]. For a very slow decoupling scenario with  $\delta_{dec} \approx 0.3 \cdot 10^{-5}$  we require  $\epsilon \approx 1.7$ .

For  $r_i = 4r_0$  we get  $\delta \propto 1/T^2$ , which quite certainly is too high. For  $r_i = 2r_0$  we find  $\epsilon = \sqrt{5} - 1 \simeq 1.24$ , which is too low. A good compromise is given by  $r_i \approx 3r_0$ , which provides a fairly good fit for moderately slow decoupling:

$$r_i = 3r_0 \rightarrow \delta \propto \frac{1}{T^{\sqrt{7}-1}} \simeq \frac{1}{T^{1.65}} \quad (95)$$

The value  $r_i = 3r_0$  is interesting, because for this value the critical mass-energy density in the standard cosmological model, which is given by  $\rho_c = 3H^2/(8\pi)$  is exactly equal to the energy density of the holostar, which is given by  $\rho = 1/(8\pi r^2)$ .

We find that the evolution of the density fluctuations requires  $r_i \approx r_0$  and thus  $H \approx 1/r$ . However, we cannot expect highly accurate results from the above treatment. In the holostar universe it is not possible to neglect the effects of the pressure. In order to incorporate pressure into the treatment above, we need to know how the anisotropic pressure will manifest itself in the co-moving

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<sup>34</sup>This matter density corresponds to  $\Omega_m \approx 0.26$  or roughly 1.5 nucleons per cubic meter at a critical density  $\rho_c$  determined from a Hubble value of  $H \simeq 71 km/s/Mpc$ .

frame. In [21] universes with anisotropic pressure were studied. The authors found, that an anisotropic pressure will appear as an isotropic pressure for the observer co-moving with the cosmic fluid. The isotropic pressure in the co-moving frame,  $P$ , is related to the anisotropic pressure components as  $P = (P_r + 2P_\theta)/3$ . If this relation is applied to the holostar, we find  $P = -1/(24\pi r^2) = -\rho/3$ . Except for the sign this is the pressure of normal radiation. In [30]  $P = 1/(24\pi r^2)$  was found for the holostar in thermodynamic equilibrium. This discrepancy clearly demonstrates, that there is an open problem with respect to the right sign of the pressure in the holostar solution with respect to the co-moving frame. Nonetheless, the sign in the pressure shouldn't influence the square of the velocity of sound, which is the relevant parameter for the density evolution in a Robertson Walker universe including pressure. Therefore I make the ansatz, that the density fluctuations in the holostar after decoupling evolve just as the density fluctuations of a radiation dominated universe. The evolution equation (85) then has to be replaced by:

$$\ddot{\delta} + 2\frac{\dot{r}}{r}\dot{\delta} - 4\pi\rho\frac{8}{3}\delta = 0 \quad (96)$$

If the above equation is expressed in terms of  $r_i/r_0$ , the only difference to the dust case in equation (89) is to replace  $r_i/r_0 \rightarrow (8r_i)/(3r_0)$ , so that the exponent in equation (93) is given by:

$$\epsilon = -1 \pm \sqrt{1 + \frac{16r_i}{3r_0}} \quad (97)$$

In order to achieve an exponent of roughly 1.65 we need  $1 + 16r_i/(3r_0) \approx 7$ , which requires

$$\frac{r_i}{r_0} \approx \frac{9}{8} \quad (98)$$

Note, that the above value is very close to unity. For  $r_i = r_0$  we find

$$\delta \propto \frac{1}{T^{\sqrt{19/3}-1}} \simeq \frac{1}{T^{1.52}} \quad (99)$$

which is a good compromise between models assuming instantaneous decoupling ( $\epsilon \approx 1.4$ ) or very slow decoupling ( $\epsilon \approx 1.7$ ).

Equation (97) is quite sensitive to the value of  $r_i/r_0$ . It seems quite clear that  $r_i$  cannot possibly lie outside the range  $0.5r_0 < r_i < 2r_0$ . If the temperature at decoupling was  $4900K$  and decoupling took place very fast, the best fit would be  $r_i/r_0 \simeq 0.9$ . For non-geodesic motion (after decoupling) one expects that the decoupling temperature will be lower, somewhere in the range between  $3500$  and  $4900K$ . A lower decoupling temperature and non-geodesic motion both require

a higher exponent in equation (93), so that  $r_i \simeq r_0$ . For non-adiabatic (i.e. not instantaneous) decoupling the baryonic density fluctuations at decoupling have been estimated to be lower (see for example [35]), placing  $\delta_{dec}$  in the range  $0.3 \cdot 10^{-5} < \delta_{dec} < 3 \cdot 10^{-5}$ , in which case  $r_i/r_0$  should be very close to unity.

Whatever the exact value of  $r_i/r_0$  might be, we are drawn to the conclusion that the massive particles making up the matter in our universe must have moved nearly geodesically from  $r_i \approx r_0$ . This is very difficult to believe without further evidence.<sup>35</sup>

An accurate estimate for the exponent in equation (93) and therefore an accurate prediction for the Hubble-value can only be made if the true equations of motion of massive particles in the holostar space-time, including pressure, are used. Furthermore at distances  $r \approx r_0$  the discreteness of the geometry and quantum effects will come into play. A detailed analysis of the motion of particles in the holostar, including the region close to its center, must be left to future research.

## 2.19 On the angular correlation of the microwave background radiation

As has already hinted in footnote 35 the experimentally determined ratio  $r_i/r_0 \approx 1$ , that appears as an independent parameter in equation (83) for the Hubble value only makes sense, if the particles in the universe as we see it today have originated from a few Planck-distances from the holostar's center. The most probable scenario is that a (presumably massive) precursor particle was created close to the center and then moved out on a nearly geodesic trajectory henceforth. Nearly geodesic motion through the hot interior region of the holostar is only conceivable for a massive particle of roughly Planck mass, which only interacts gravitationally. Somewhere on its trajectory, not too far from the radial coordinate value where the temperature has dropped below the electro-weak unification scale, the particle then must have decayed into ordinary matter, endowing the final products of the decay process (electrons, protons, neutrons and neutrinos - or more generally quark/leptons) with the high momentum gathered on its outward track. This scenario is quite an extravagant claim, for which it would be quite helpful to have another - independent - verification. The missing angular correlation of the microwave background radiation at large angles provides such a means.

The recent WMAP data [37] have revealed, that there is practically no correlation between the fluctuations in the microwave background radiation at an

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<sup>35</sup>Note also that  $r_i \approx r_0$  indicates, that the particles must have had extremely high radial velocities at decoupling: With the - incorrect - assumption that the number of particles remains constant during the expansion the order of magnitude of the relativistic  $\gamma$ -factor at decoupling can be roughly estimated:  $\gamma_{dec} \approx \sqrt{r_{dec}/r_i} \approx 1.7 \cdot 10^{27}$ . Such high velocities at decoupling are only conceivable, if the massive particles that constitute the matter of the universe today have truly originated from  $r_i \approx r_0$ . This possibility is discussed in section 2.19.

angular separation larger than approximately  $60^\circ$ , which corresponds to roughly 1 radian. This feature can be quite effortlessly explained by the motion of massless and massive particles in the holostar:

As can be seen from equation (23) particles that were emitted just a few Planck distances from the center of a holostar have a limited angular spread. According to the scenario proposed beforehand let us assume, that a massive, uncharged particle with a fairly long lifetime is created with not too high a tangential velocity  $\beta_i$  close to the center of the holostar. Its maximum angular spread can be calculated:

$$\varphi_{max} = \beta_i \sqrt{\frac{r_i}{r_0}} \int_0^1 \frac{dx}{\sqrt{1-x(1-\beta_i^2(1-x^2))}} = \beta_i \sqrt{\frac{r_i}{r_0}} \xi(\beta_i^2) \quad (100)$$

$\xi = \xi(\beta_i^2)$  is the value of the definite integral in the above equation for the value of  $\beta_i$  at the turning point of the motion. As remarked in section 2.5 its value lies between  $1.4 < \xi \leq 2$ . The above equation gives an implicit relation for  $\beta_i^2$ , which is the mean velocity  $\beta_i$  of the particle at the radial coordinate position of its turning point of the motion (or rather the radial coordinate position  $r_i$  where the motion of the particle has become nearly geodesical). Equation (100) can be solved iteratively for  $\beta_i^2$ , whenever the maximum angular spread  $\varphi_{max}$  and the ratio  $r_i/r_0$  is known.

$\varphi_{max}$  and  $r_i/r_0$  can be determined experimentally. However, there is a subtlety involved in the experimental determination of  $r_i/r_0$  from the characteristics of the expansion. In the derivation of equation (83) for the Hubble-value I have assumed, that the motion started out from  $r_i$  at rest. This is unrealistic. At  $r_i \approx r_0$  there is an extremely high temperature, so that even a particle of nearly Planck mass will have an appreciable velocity at its (true) turning point of the motion,  $r_i$ . Therefore the ratio  $r_i/r_0$  in the Hubble equation (83) doesn't refer to the true turning point of the motion  $r_i$ , but rather to a fictitious "zero-velocity" turning point  $\tilde{r}_i$ , which describes the radial part of motion far away from the turning point. Both values are related: In section 2.5 it has been shown, that the radial part of the motion of a particle with an appreciable tangential velocity at its turning point is nearly identical to the motion of a particle that started out from a somewhat smaller radial coordinate value, whenever  $r \gg r_i$ . The relation between the true turning point of the motion and the apparent turning point of the motion is given by:

$$r_i = \gamma_i^2 \tilde{r}_i \quad (101)$$

where  $\gamma_i$  is the special relativistic  $\gamma$ -factor of the particle at its true turning point of the motion  $r_i$ .

The experimentally determined ratio  $r_i/r_0$  in the Hubble equation (83) or in the equation for the density evolution (96) therefore rather refers to  $\tilde{r}_i/r_0$ ,

whereas  $r_i$  in equation (100) rather refers to the true turning point of the motion. In section 2.18 this ratio has been estimated as  $\tilde{r}_i/r_0 \approx 1$ , if the density perturbations found in the microwave background radiation  $\delta \approx 10^{-5}$  evolve according to equation (93) in combination with equation (97) to the value observed today. Let us denote the ratio  $\tilde{r}_i/r_0$  with  $\kappa$ :

$$\kappa = \frac{\tilde{r}_i}{r_0} = \frac{1}{\gamma_i^2} \frac{r_i}{r_0} \quad (102)$$

The equation for the maximum angular spread then reads:

$$\varphi_{max}^2 = \beta_i^2 \xi^2 \frac{r_i}{r_0} = \beta_i^2 \gamma_i^2 \xi^2 \kappa \quad (103)$$

so that:

$$\beta_i^2 \gamma_i^2 = \frac{\beta_i^2}{1 - \beta_i^2} \simeq \frac{\varphi_{max}^2}{\xi^2 \kappa} \quad (104)$$

Equation (104) has to be solved for  $\beta_i^2$  (or alternatively for  $\beta_i^2 \gamma_i^2$ ). Once  $\beta_i \gamma_i$  is known, the true turning point of the motion can be determined:

$$r_i = \gamma_i^2 \tilde{r}_i = (1 + \beta_i^2 \gamma_i^2) \kappa r_0 \quad (105)$$

where the relation  $\gamma^2 = 1 + \beta^2 \gamma^2$  was used.

For  $\varphi_{max} = 60^\circ = \pi/3 \approx 1$  the maximum angular spread is nearly unity (in radians). For  $\kappa = 1$  we find  $\xi \approx 1.77$ . With these values

$$\beta_i^2 \gamma_i^2 \simeq 0.319 \quad (106)$$

so that

$$\beta_i^2 = \frac{\beta_i^2 \gamma_i^2}{1 + \beta_i^2 \gamma_i^2} \simeq 0.242 \quad (107)$$

and

$$\frac{r_i}{r_0} \simeq 1.319 \quad (108)$$

If we know  $\beta\gamma$  of a massive particle at the radial position  $r_i$ , where the particle's motion has become nearly geodesical (i.e. the particle is effectively decoupled from the other particles), we can determine the mass of the particle. The momentum of any massive particle with mass  $m_0$  is given by

$$p = \beta\gamma m_0 \quad (109)$$

The massive particle will be immersed in a very hot radiation bath of ultra-relativistic or zero rest mass particles. Most likely it was created at the hottest possible spot of the holostar, i.e. somewhere in the region  $r_0/2 < r < r_0$ .<sup>36</sup> During its "movement" from the "point" of its creation to its "point" of decoupling,  $r_i \approx r_0$ , the radiation will have imprinted its momentum on the massive particle  $m_0$ . In the holostar the mean momenta  $\bar{p}_\gamma$  of the ultra-relativistic particles are proportional to the interior radiation temperature, given by equation (51).

$$\bar{p}_\gamma = sT_\gamma = \frac{s}{4\pi} \frac{\hbar}{\sqrt{r_i r_0}} \quad (110)$$

In [30] it is shown, that the factor of proportionality,  $s$ , is equal to the entropy per particle (which is slightly larger than  $\pi$  for a great variety of circumstances). Putting all equations together and using the relation  $r_0^2 \approx 4s/\pi$  at the Planck-energy proposed in [28] we find:

$$m_0 \simeq \frac{1}{8} \sqrt{\frac{s}{\kappa\pi}} \frac{\sqrt{\hbar}}{\beta_i \gamma_i^2} \quad (111)$$

With  $\kappa = 1$  and  $\varphi_{max} = 1$  the mass of the "precursor" particle  $m_0$  is given by:

$$m_0 \simeq 0.201 m_{Pl} \quad (112)$$

if  $s = 3.37174$  is used. This value is the mean entropy per ultra-relativistic particle in the supersymmetric phase expected to be present at the center of the holostar, where the fermionic and bosonic degrees of freedom are equal and the chemical potentials of bosons and fermions are proportional to the temperature and opposite to each other (for a more detailed discussion see [30]).

The particle has a mass roughly a fifth of the Planck mass. Due to  $\bar{p}_\gamma = sT_\gamma = \sqrt{s/\pi}/4 \approx 0.561 m_0 \approx 0.113 m_{Pl}$  the production of this particle will be somewhat inhibited at  $r = r_i$ , as an energy of  $E = \sqrt{\bar{p}_\gamma^2 + m_0^2} \approx 0.231 m_{Pl}$

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<sup>36</sup>There is good theoretical reason to believe, that  $r_0/2$  provides a universal cut-off for the region that can be occupied by any one particle. No particle will be able to enter the region bounded by the smallest possible area quant of quantum gravity which turns out to be roughly equal to  $4\pi(r_0/2)^2$  (see [28]). Therefore the smallest "separation" between two particles should be roughly equal to  $r_0$ , as far as classical reasoning can still be trusted at the Planck scale. In [30] the number of particles within a spherical concentric region has been determined as  $N = \pi/sr^2/\hbar \simeq r^2/\hbar$ , as  $s \approx \pi$ . With the experimental estimate  $r_0 \approx 2\sqrt{\hbar}$  from equation (54) we find, that  $N \simeq 1$  for  $r = r_{Pl} \approx r_0/2$ . Note also, that in [28] it has been shown, that  $r_0/2$  is the radius of the membrane of an elementary extremal holostar with zero (or negligible) mass, i.e. the smallest holostar possible.

is required. However, at  $r = r_0/2 \approx r_i/2$  there is just enough energy available to create the "precursor" particles in (in pairs) in substantial numbers and with the right momenta, either by the collision of any two massive or massless particles of the surrounding radiation bath, or - slightly more efficiently - by Unruh radiation: At a radial coordinate position  $r = r_0/2 \approx r_i/2.6$  the local radiation temperature  $T_\gamma$  will be up by a factor of  $\sqrt{2.6}$  with respect to the temperature at  $r_i$ , so that the mean momentum of the radiation will be given by  $\bar{p}_\gamma \approx 0.183 m_{Pl}$ . The mean energy of the massive particle at  $r_0/2$  will be higher as well, but not with the same factor as the radiation, because of its non-negligible rest-mass. We find:  $E_0 \approx \sqrt{\bar{p}_\gamma^2 + m_0^2} \approx 0.272 m_{Pl}$ . The mean momentum of the radiation quanta doesn't yet suffice to produce the precursor particles efficiently in pairs, at least not with the right momentum. However, the mean energy of the two radiation quanta is almost large enough to produce two precursor particles with zero momentum. Presumably the most efficient production of the precursor particles is via Unruh radiation between  $r_0/2 < r < r_0$ . The Unruh-temperature at  $r = r_0/2$  will be twice the radiation temperature (see section 2.24). The energy of a particle produced by Unruh radiation therefore is  $E \approx 0.366 m_{Pl} \approx 1.35 E_0$ . The production of the precursor particles by Unruh radiation should be quite efficient, even if the high chemical potential of the particles,  $\mu = 1.353 T_\gamma$ , in the supersymmetric phase is taken into account.<sup>37</sup>

The mass of the "precursor" particle  $m_0$  only depends moderately on  $\kappa$ . If  $\kappa$  lies in the range  $0.5 < \kappa < 4$  we find the following mass-range for  $m_0$ :

$$0.164 < m_0 < 0.243 \quad (113)$$

in Planck units.

Below a table for the masses and the tangential velocity  $\beta_i$  of the precursor particle is given for several values of  $\kappa$  with  $\varphi_{max} = 56.8^\circ$ :

$\kappa$	$\beta_i$	$\xi$	$m_0$
1/2	0.647	1.65	0.165
3/4	0.553	1.72	0.188
1	0.490	1.77	0.201
3/2	0.405	1.83	0.218
2	0.353	1.87	0.228
5/2	0.316	1.89	0.234
3	0.288	1.91	0.238
4	0.250	1.93	0.243

If equation (96) for the density evolution (with pressure) is correct,  $\tilde{r}_i$  should lie in the range  $3/4 < \tilde{r}_i/r_0 < 5/4$ , so that the bound for the mass will be roughly

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<sup>37</sup>For the relevance of chemical potentials and the thermodynamics of highly relativistic matter states see [30].

$0.19 < m < 0.21$  in Planck units. A value of  $\tilde{r}_i$  that lies this range also fits better with the current measurements of the Hubble-constant (see section 2.20).

We find, that if the current expansion rate of the universe and the evolution of the density perturbations after decoupling can be (approximately) described by the holostar solution, one requires a new particle. Lets call it the "preon".<sup>38</sup> Its properties can be quite accurately inferred from the above discussion: Its mass should be less than one quarter of the Planck mass. Its exact value shouldn't lie too far outside the mass range given by equation (113). The particle should interact only gravitationally and should be relatively long lived ( $\tau \approx 10^{-6} \dots 10^{-2} s$ ), so that it can survive up to electro-weak transition (or slightly longer).

It is possible to estimate the mass-energy of the preon by another, independent argument. According to the discussion in footnote 36 the proper volume occupied by a single particle at the holostar's center is given by

$$V_1 = \int_0^{r_0/2} dV = \frac{8\pi}{7\sqrt{2}} \left(\frac{r_0}{2}\right)^3 \approx \frac{8\pi}{7\sqrt{2}} V_{Pl} \quad (114)$$

On its way outward the volume available to the preon will become larger, which enables it to decay into lighter particles, such as neutrons, protons and electrons. The decay is expected to conserve mass-energy locally. What will the energy-density of the stable decay products be at large  $r$ -values? To simplify the calculations it is convenient to consider a thought-experiment, in which the proper length of the expanding volume remains constant in the radial direction. This thought-experiment simplifies the calculations as we don't have to take into account any changes of the internal energy in the expanding volume due to the radial pressure.<sup>39</sup> Any motion that leaves the internal energy of a spherically outmoving volume unaffected requires that the volume develops as the proper surface area of an expanding spherical shell with constant proper thickness, i.e.  $V \propto r^2$ .

At the radial position  $r = 9.18 \cdot 10^{60} r_{Pl}$ , which corresponds to the radius of the observable universe today, the volume  $V_1$  will have expanded to:

$$V_{today} = \left(\frac{r}{r_{Pl}}\right)^2 V_1 \simeq 1.0 \cdot 10^{18} m^3 \quad (115)$$

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<sup>38</sup>The "pre" stands for "precursor". Unfortunately the term "geon", which might be considered even more appropriate (the syllable "ge" then could refer to "genesis" as well as "geometric") has already been taken.

<sup>39</sup>Keep in mind that the actual motion of the preon is different from the thought-experiment. However, the thought-experiment allows us to neglect the effects of the negative radial pressure without changing the physical results obtained from a more realistic treatment. If the pressure-effects are included in the total energy-balance, the actual (geodesic) motion of the preon yields the same results as the method of "constant internal energy" in the thought-experiment.

Assuming local mass-energy conservation and assuming that the mass-energy of the preon ends up predominantly in neutrons (i.e. no significant dark matter component; energy-contributions of electrons, neutrinos and photons negligible with respect to the baryons), the total number of neutrons within this volume will be given by:

$$N_n = \frac{E_{preon}}{m_p} \approx 1.68 \cdot 10^{18} \quad (116)$$

if the mass-energy of the preon is assumed to be  $E_{preon} = p_\gamma(r_0) \simeq \sqrt{s/\pi} m_{Pl}/8 = 0.13 m_{Pl}$ . From equations (115, 116) the number-density of nucleons in the universe today can be estimated as:

$$n_n = 1.68 \frac{1}{m^3} \quad (117)$$

amounting to roughly 1.7 nucleons per cubic meter. This is very close to the number-density of nucleons in the universe derived from the total matter-density determined by WMAP assuming no significant dark matter component,  $n_n = 1.48/m^3$  (see section 2.20). Therefore a preon-mass in the range between 0.1 to 0.2 Planck masses is quite consistent with the findings in the observable universe today.

The assumption that the preon eventually decays into nucleons at a temperature slightly below the nucleon rest-mass, enabled us to give a - very crude - estimate for the absolute value of the density contrast  $\delta$  at the time of baryogenesis. Astoundingly this crude estimate fits quite well with the experimentally determined values of the density contrast today  $\delta_{today} \approx 1$  and at the time of decoupling  $\delta_{dec} \approx 10^{-5}$ :

The red-shift  $z_b$  where the mean energy of the radiation is equal to the nucleon rest-mass is given as:

$$z_b \approx \frac{m_p/3.37}{T_{CMBR}} \approx 1.19 \cdot 10^{12} \quad (118)$$

At this redshift the number-density of the nucleons  $n_b$  in the holostar will be higher than today by a factor of  $z_b^4$ , i.e.

$$n_b \approx \frac{1.5}{m^3} z_b^4 \approx \frac{3 \cdot 10^{48}}{m^3} \quad (119)$$

which corresponds to a matter-density of  $\rho_b \approx 5 \cdot 10^{21} kg/m^3$ . This value is roughly a factor of thousand higher than the typical neutron star density and four orders of magnitude higher than the typical density of stable nuclei.

If the nucleons in our universe originate from the preon at roughly this redshift, one would expect a density-contrast  $\delta_b$  on the order of the nucleon to preon mass at this time. With a preon mass  $m_{preon} \approx 0.15m_{Pl}$  we find

$$\delta_b \approx \frac{m_p}{m_{preon}} \approx 5 \cdot 10^{-19} \quad (120)$$

As has been shown in section 2.18 the density contrast evolves as a power-law with respect to the redshift. For  $r_i = r_0$  the exponent is given by  $\epsilon = \sqrt{19/3} - 1 \simeq 1.517$ , so that:

$$\delta \propto \frac{1}{z^{1.517}} \quad (121)$$

Therefore we can "predict" the density-contrast at any redshift  $z < z_b$  from the density-contrast at the time of baryogenesis. We find:

$$\delta_{dec} = \delta_b \left( \frac{z_b}{z_{dec}} \right)^{1.517} \approx 1.2 \cdot 10^{-5} \quad (122)$$

with  $z_{dec} \approx 1800$  and

$$\delta_{today} = \delta_b \left( \frac{z_b}{1} \right)^{1.517} \approx 1.05 \quad (123)$$

So far the maximum angular correlation distance of the microwave background radiation ( $\varphi_{max} \approx 60^\circ$ ) has been put in by hand, as determined from the observations. It would be nice, if this value could be derived by first principles. There appears to be a way to do this. For this purpose let us consider the angular spread of zero mass particles in the holostar. The maximum angular spread for a photon emitted from  $r_i$  is given by:

$$\varphi_{max} = \frac{\sqrt{\pi}}{3} \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{5}{6})} \sqrt{\frac{r_i}{r_0}} \simeq 1.4022 \sqrt{\frac{r_i}{r_0}} \quad (124)$$

Let us consider photon-pair production (or the production of any other massless particle in pairs) by Unruh radiation. In order to produce a photon pair with a mean momentum equal to the local radiation temperature  $T_\gamma$ , the Unruh-temperature  $T_U$  should be twice the radiation temperature. In section 2.24 the following relation between radiation temperature and Unruh-temperature will be derived:

$$\frac{T_U}{T_\gamma} \simeq \frac{r_0}{r} \quad (125)$$

In order for the Unruh temperature to be twice as high as the radiation temperature we need  $r_i = r_0/2$ . Note that according to [28]  $r_i = r_0/2 \approx r_{Pl}$  is the position of the membrane of an "elementary" extreme holostar, which appears to be the smallest possible "size" for a (classical) holostar. There are some good reasons to identify an "elementary" extreme holostar (with nearly zero mass) with an elementary particle. As a fundamental particle cannot be composite, i.e. there cannot be any particle "within" a fundamental particle, and as  $4\pi(r_0/2)^2$  is very close to the smallest area quantum of quantum gravity, it is very probable that the region  $r < r_0/2$  is not accessible to any particle, probably not even well defined. Therefore Unruh creation of photon pairs occurs efficiently just at the most central conceivable location that is available for a "particle" (or more generally: a discrete geometric entity) within a large holostar. If we insert the ratio  $r_i/r_0 = 1/2$  into equation (124) we get:

$$\varphi_{max} = \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{5}{6})} \sqrt{\frac{\pi}{18}} \simeq 0.9915 = 56.81^\circ \quad (126)$$

Voilà.

Finally it should be noted, that  $\varphi_{max} \approx 1$  makes the "expansion" of particles that move radially outward in the holostar nearly equal for the radial and the tangential directions:  $\delta l_\perp = r\varphi_{max}$ , whereas  $\delta l_r \approx r$ .

## 2.20 Estimating cosmological parameters from the radiation temperature

In this section the total local mass-energy density, the local Hubble value, the radial coordinate value  $r$  and the proper time  $\tau$  in a "holostar universe" will be determined from the temperature of the microwave background radiation.

The total energy density in the holostar universe can be determined from the radiation temperature, whenever  $r_0^2$  is known. It is given by equation (52):

$$\rho = \frac{2^5 \pi^3 r_0^2}{\hbar^4} T^4 \quad (127)$$

There is some significant theoretical evidence for  $r_0^2 \approx 4\sqrt{3/4}\hbar$  at the low energy scale. However  $r_0^2 = 4\hbar$  or a value a few percent higher than  $4\hbar$  might also be possible (for a somewhat more detailed discussion see [28]). With  $r_0^2 = 4\sqrt{3/4}\hbar$  and  $T_{CMBR} = 2.725K$  we find:

$$\rho = 2.425 \cdot 10^{-27} \frac{kg}{m^3} = 1.450 \cdot \frac{m_p}{m^3} = 4.702 \cdot 10^{-124} \rho_{Pl} \quad (128)$$

This is almost equal to the total matter-density of the universe determined by WMAP [37]:

$$\rho_{WMAP} = 2.462 \cdot 10^{-27} \frac{kg}{m^3} \quad (129)$$

From the matter density the radial coordinate position  $r$  within the holostar can be determined.

$$r = \frac{1}{\sqrt{8\pi\rho}} = 9.199 \cdot 10^{60} r_{Pl} = 1.575 \cdot 10^{10} ly \quad (130)$$

This value is quite close the radius of the observable universe.

The local Hubble-constant can be determined from the matter density via equation (83):

$$H = \sqrt{8\pi\rho \frac{r_0}{r_i}} \quad (131)$$

With  $r_i/r_0 = 1$  and the matter density determined beforehand we find:

$$H = 2.021 \cdot 10^{-18} \left[\frac{1}{s}\right] = 62.36 \left[\frac{km/s}{Mpc}\right] \quad (132)$$

The Hubble-constant comes out quite close to the value that is used in the concordance model by the WMAP-group [37] with  $H = 71 km/s/Mpc$ . This is an encouraging result. Note, however, that the value of the Hubble constant is model-dependent. It is possible to relate the Hubble value of the various standard cosmological models to the Hubble value of the holostar solution via the mass-density. In the standard cosmological models we find:

$$\rho_m = \Omega_m \rho_c = \Omega_m \frac{3H_s^2}{8\pi} \quad (133)$$

For the holostar the mass-density is given by

$$\rho_m = \frac{r_i}{r_0} \frac{H_h^2}{8\pi} \quad (134)$$

Setting the mass densities equal, the local Hubble-values can be related:

$$H_h^2 = \Omega_m \frac{3r_0}{r_i} H_s^2 \quad (135)$$

Both values are equal, if  $\Omega_m = r_i/(3r_0)$ . This is almost the case for  $\Omega_m \approx 0.26 \approx 1/4$  according to WMAP and  $r_i/(3r_0) \approx 1/3$  as determined in section 2.18. This result is quite robust. If we determine  $H_h$  from  $H_s$  via equation

(135) for various combinations of  $\Omega_m$  and  $H_s$  that have been used in the past<sup>40</sup> we get the result of equation (132) with an error on the order of a few percent.

From the local Hubble-value determined in (132) the proper time  $\tau$  can be derived:

$$\tau = \frac{1}{H} = 9.180 \cdot 10^{60} t_{Pl} = 1.57 \cdot 10^{10} y \quad (136)$$

This is somewhat larger than the result recently announced by the WMAP group,  $t = 1.37 \cdot 10^{10} y$ .

If we set  $r_i/r_0 = 3/4$  we find an almost perfect agreement of the holostar's local Hubble value  $H_h = 71.85$  (km/s)/Mpc and the current proper time  $\tau = 13.6 Gy$  with respect to the values determined by WMAP [37] ( $H = 71$  (km/s)/Mpc and  $t = 13.7 Gy$ ).

The holostar solution is quite compatible with the recent findings concerning the large scale structure and dynamics of the universe. The recent WMAP results are reproduced best by setting  $r_i/r_0 = 3/4$ . From the evolution of the density perturbations (see section 2.18) one would rather expect  $r_i \simeq r_0$ . It is quite clear that  $r_i/r_0$  cannot lie very much outside the range  $3/4 < r_i/r_0 < 1$ , which corresponds to an age of the universe in the range between 13 – 16 Gy.

Note that the above age comes close to the ages of the oldest globular clusters. If  $r_i/r_0$  is chosen larger than unity, the proper time  $\tau$  in the holostar universe will be larger. For  $r_i/r_0 \approx 2$  the holostar universe can easily accommodate even the old estimates for the ages of the globular clusters, which not too far ago have been thought to lie between 13 to 19 billion years. The problem with such an assignment is, that the Hubble value comes out far too low. For  $r_i = 2r_0$  we find  $H \approx 44$  (km/s)/Mpc, which appears incompatible with today's experimental measurements, even if the large systematic errors in calibrating the cosmological distance scale are taken into account.

From a theoretical point of view (see the discussion in section 2.24, where the CMBR-temperature is interpreted as Unruh-temperature) I find  $r_i = r_0$  the most preferable choice. However, this choice is based on equation (151), which assumes  $(T_i/m)(\sigma/\hbar) = 1$ . This relation is based on classical reasoning (at the Planck-energy scale) so that one should expect some moderate adjustment due the quantum nature of space-time at this scale. Therefore it is too early to make a definite numerical prediction for the ratio  $r_i/r_0$ . If  $r_i/r_0$  can be pinned down theoretically we are in the very much desirable position to make a precise prediction for the Hubble value, which should be an order of magnitude better than today's measurement capabilities. Therefore any significant advances in the understanding of the motion of particles in the holostar universe could be of high practical value for the development of a "high precision cosmology" in the near future.

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<sup>40</sup>Some years ago  $\Omega_m \approx 0.3$  and  $H_s \approx 65 km/s/Mpc$  was a common estimate.

## 2.21 Local matter distribution and self-similarity

For a very large holostar of the size of the universe, its outer regions will consist of low-density matter, which should be quite comparable in density and/or distribution to the matter we find in our universe. Such matter can be expected form local hierarchical sub-structures comparable to those found in our universe, as long as the scale of the sub-structures remains compatible with the overall mass-energy distribution  $\rho \propto 1/r^2$  of the holostar.<sup>41</sup>

On not too large a scale some local regions might collapse, leaving voids, others might expand, giving rise to filamental structures. However, any local redistribution of mass-energy has to conserve the total gravitational mass of the holostar, its total angular momentum and - quite likely - its entropy, i.e. its total number of particles. These exterior constraints require that regions of high matter-density must be accompanied by voids.

Furthermore one can expect, that the local distribution of matter within a holostar will exhibit some sort of self-similarity, i.e. the partly collapsed regions should follow a  $1/r^2$ -law for the local mass-density. This expected behavior is quite in agreement with the observations concerning the mass-distribution in our universe: The flat rotation curves of galaxies, as well as the velocity dispersion in galaxies and clusters of galaxies hint strongly, that the matter distribution of the local matter in galaxies and clusters follows an  $1/r^2$ -law. As far as I know, there has been no truly convincing explanation for this apparently universal scaling law, so far.

## 2.22 Some remarks about the frames of the asymptotic and the co-moving observer

Most of the discussion about the properties of the holostar has been in the frame of the asymptotic observer, who is at rest in the  $(t, r, \theta, \varphi)$ -coordinate system.

If the holostar is to serve as a model for an expanding universe, one must interpret the phenomena from the frame of the co-moving observer, who moves nearly geodesically on an almost radial trajectory through the low density outer regions of the holostar.

The frames of the co-moving and the asymptotic observer are related by a Lorentz boost in the radial direction. Due to the small tidal acceleration in the holostar's outer regions, the extension of the local Lorentz frames can be fairly large. The proper acceleration experienced by the co-moving observer, if there is any, should be very low in the regions which have a density comparable to the density that is observed in the universe at the present time.

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<sup>41</sup>The largest scale for significant local deviations from the mass-density is given by the proper circumference at radial position  $r$ , i.e. roughly the Hubble length.

The holostar's interior space-time is boost-invariant in the radial direction, i.e. the stress-energy tensor is unaffected by a radial boost. The co-moving observer moves nearly radially for  $r \gg r_i$ . His radial  $\gamma$ -factor grows as the square root of his radial coordinate value, whereas his tangential velocity goes rapidly to zero with  $1/r^3$ . The radially boosted co-moving observer therefore will see exactly the same total stress-energy tensor, and thus the same total energy density as the observer at rest in the  $(t, r, \theta, \varphi)$ -coordinate system. The above statement, however, only refers to the total energy density. It is not a priori clear if the individual contributions to the mass-energy, i.e. massive particles and photons, have the same  $r$ -dependence as the total energy density.

Let us first consider the case of massive particles. The observer at rest in the coordinate system measures an energy density  $\rho$  of the massive particles, which is proportional to  $1/r^2$ . A factor of  $1/r^{5/2}$  comes from the number density given in equation (71), a factor of  $r^{1/2}$  from the special relativistic  $\gamma$ -factor that must be applied to the rest mass of the particles according to equation (74).

From a naive perspective (neglecting the effects of the pressure) it appears as if the co-moving observer and the observer at rest in the  $(t, r, \theta, \varphi)$ -coordinate system disagree on how the energy and number densities of massive particles develop with  $r$ . Because of the highly relativistic motion of the co-moving observer, the observer at rest in the  $(t, r, \theta, \varphi)$ -coordinate system will find that the proper volume of any observer co-moving with the massive particles is Lorentz-contracted in the radial direction. Therefore the co-moving observer will measure a larger proper volume, enlarged by the radial  $\gamma$ -factor, which is proportional to  $\sqrt{r}$ . If we denote the volume in the frame of the co-moving observer by an overline, we find  $\overline{V} \propto \gamma r^{5/2} \propto r^3$ . As long as the massive particles aren't created or destroyed, the number-density of the massive particles in the co-moving frame therefore must scale as  $1/r^3$ . Furthermore, for the co-moving observer the neighboring massive particles are at rest to a very good approximation.<sup>42</sup> Therefore the mass-energy density in the co-moving frame should be nothing else than the (presumably) constant rest-mass of the particles multiplied by their number-density. From this it follows, that the mass-energy density of the massive particles should scale as  $1/r^3$  as well.

This naive conclusion, however, is false. It does not take into account the energy change in the co-moving volume due to the radial pressure. Any radial expansion in the co-moving frame affects the internal energy. Due to Lorentz-elongation in the co-moving volume the radial thickness  $\overline{l}_r$  of the expanding shell develops proportional to  $r$  in the co-moving frame. From this the internal energy-change  $\delta E$  in the shell can be calculated for a small radial displacement  $\delta r$ . We find:

$$\overline{\delta E} = -\overline{P}_r A \overline{\delta l}_r \propto \frac{\delta r}{2} \quad (137)$$

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<sup>42</sup>The tidal acceleration is negligible; the extension of the local Lorentz frame of the observer is nearly equal to the local Hubble length, i.e. of the order the current "radius" of the universe,  $r$ .

with  $\overline{P}_r = P_r = -1/(8\pi r^2)$  and  $\overline{A} = 4\pi r^2$ . Note that the radial pressure in the co-moving frame is exactly equal to the radial pressure in the coordinate frame due to the boost-invariance of the stress-energy tensor in the radial direction.

From equation (137) the radial dependence of the internal total energy in the co-moving frame follows:

$$\overline{E} \propto r$$

We have already seen that the co-moving volume develops as  $\overline{V} \propto r^3$ , so that the energy density of the massive particles in the co-moving frame, taking the pressure into account, develops exactly as in the coordinate frame, i.e.  $\overline{\rho} \propto \overline{E}/\overline{V} \propto 1/r^2$ .

It is quite obvious that such a dependence is not compatible with the assumption that both the rest-mass and the number of the massive particles in the co-moving volume remain constant. Either the rest-mass of the massive particles must increase during the expansion or new particles (massive or radiation) have to be created by the negative pressure. There is no indication whatsoever that the rest mass of the nucleon or the electron have changed considerably during the evolution of the universe, at least for temperatures at and below the time of nucleosynthesis. Therefore we cannot avoid the conclusion that the negative pressure has the effect to create new particles. Particle creation in an expanding universe is not new. It is one of the basic assumptions of the venerable steady-state model of the universe. Furthermore particle production via expansion against a negative pressure is a well known phenomenon from the inflationary equation of state. There are differences. Whereas the isotropic negative pressure of the inflationary phase keeps the energy-density in the expanding universe constant during the expansion, the energy-density in the holostar develops as  $1/r^2$ , because the negative pressure only acts in one of three spatial directions. As a result the particle creation rate in the holostar is quite low at the present time: Roughly one neutron per cubic kilometer every 10 years is required.

The above analysis indicates that the energy density of the massive particles in the co-moving frame should be proportional to  $1/r^2$ , exactly as in the coordinate frame. A radial boost not only leaves the total energy density unaffected, but also the respective energy densities of the different particle species.<sup>43</sup> With the reasonable assumption that the rest mass of the massive particles is constant (at least for temperatures well below the rest mass of the particles) the number densities must also evolve according to  $1/r^2$ .

Let us now discuss the number- and energy densities of the zero rest-mass particles in the two frames. This problem is closely related to the question

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<sup>43</sup>This argument is not water tight. If the particles produced by the negative pressure are different from the (massive) particle species that catalyze their production, there might be a redistribution of mass-energy between the different species during the expansion.

whether the co-moving observer experiences a different radiation temperature than the observer at rest in the  $(t, r, \theta, \varphi)$ -coordinate system. The standard argument for the red-shift of radiation in an expanding Robertson-Walker universe is, that the wavelength of the radiation is stretched proportional to the expansion.<sup>44</sup> This gives the known  $T \propto 1/r$ -dependence for the radiation temperature in the standard cosmological models. If this argument is applied to the holostar, one would expect that the  $T \propto 1/\sqrt{r}$ -law (in the frame of the observer at rest) is transformed to a  $\bar{T} \propto 1/r$ -law in the frame of the co-moving observer, due to Lorentz-contraction (or rather elongation) of the photon wavelength.

But a  $\bar{T} \propto 1/r$ -law wouldn't be consistent for a small holostar, where the energy density is expected to be dominated by radiation in true thermal equilibrium. The energy density of thermalized radiation is proportional to  $T^4$ . If the only contributor to the total energy density is radiation, the radiation energy density will transform exactly as the total energy density in a radial boost. However, the total energy density is radially boost-invariant. Therefore in the radiation dominated era the energy density of the radiation in any radially boosted frame should scale as  $1/r^2$ . This however implies the  $\bar{T} \propto 1/\sqrt{r}$ -dependence, at least if the radiation is in thermal equilibrium in the boosted frame.<sup>45</sup>

In a radially boost-invariant space-time one would expect on more general grounds, that it is - in principle - impossible to determine the radial velocity of the motion, at least by direct measurements performed by the observer co-moving with the matter-flow. A  $\bar{T} \propto 1/r$ -law would imply a radiation temperature incompatible with the energy-density of the radiation, which would allow the co-moving observer to determine his radial velocity. This can be seen as follows:

Naively one would assume, that the number-density of photons in the co-moving frame is given by  $\bar{n}_\gamma = n_\gamma/\gamma \propto 1/r^2$ , due to Lorentz elongation of the co-moving volume with respect to the observer at rest. However, at a closer look one has to take into account the measurement process. Photons cannot be counted just by putting them in a box and then taking out each individual particle. As photons always move with the local speed of light, such a procedure, which would be good for massive particles, doesn't work. The right way to count photons is to place a small ideal (spherical) absorber somewhere in the space-time, count all the hits per proper time interval and relate the obtained number to the volume (or surface area) of the absorber. But this procedure requires, that the time-delay due to the highly relativistic motion of the co-moving observer

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<sup>44</sup>A more sophisticated derivation is based on the existence of a Killing vector field in a Robertson-Walker universe (see for example [41, p.101-104]). The derivation makes use of the well known fact, that the scalar product of the photon wave-vector with the Killing-vector is constant for geodesic motion of photons. This argument, however, requires the geodesic equations of motion, and therefore is only water-tight for a dust universe without significant pressure.

<sup>45</sup>If the holostar were truly static in the high temperature regime, i.e. geodesic and pressure induced acceleration cancel exactly, there is no problem. There would be no directed motion and therefore  $\bar{T} = T \propto 1/\sqrt{r}$  trivially.

has to be taken into account. The co-moving observer will count many more photons in a given standard interval of his proper time, than the observer at rest would count in the same interval. The time-dilation introduces another  $\gamma$ -factor that exactly cancels the  $\gamma$ -factor from the Lorentz contraction of the proper volume. Therefore we arrive at the remarkable result, that the number-density of photons should be the same for both observers, i.e. independent of a radial boost.<sup>46</sup>

If we had  $\bar{T} \propto 1/r$  in the co-moving frame, we would get  $\bar{\rho}_\gamma \propto \bar{n}_\gamma \bar{T} \propto 1/r^{5/2}$ , which implies  $\bar{\rho}_\gamma \propto \bar{T}^{5/2}$  in the co-moving frame. However, in thermal equilibrium  $\bar{\rho}_\gamma \propto \bar{T}^4$ . Even if the argument given above for the number-density of photons were incorrect, i.e. only the volume were Lorentz contracted and time dilation would play no role, we would have  $\bar{\rho}_\gamma \propto \bar{n}_\gamma \bar{T} \propto 1/r^3$ , implying  $\bar{\rho}_\gamma \propto \bar{T}^3$  in the co-moving frame, which wouldn't work either.

Therefore it seems reasonable to postulate, that a radial boost from the  $(t, r, \theta, \varphi)$ -coordinate system to the system of the co-moving observer should not only leave the total energy density unaffected, but also other physically important characteristics such as the thermodynamic state of the system, i.e. whether or not the radiation can be characterized as thermal. This then implies that the thermodynamic relation between energy-density and temperature for an ultra-relativistic gas should be valid in the radially boosted frame of the co-moving observer, i.e.  $\bar{\rho} \propto \bar{T}^4$ , which requires the  $\bar{T} \propto 1/\sqrt{r}$  law.

The reader may object, that any radial boost will produce a large anisotropy in the radiation temperature, as measured in the boosted frame. I.e. the radiation will be blue-shifted for photons travelling opposite to the motion of the co-moving observer and red-shifted for photons travelling in the same direction, due to "normal" Doppler shift. However, this again doesn't take into account the subtle effects of the negative radial pressure. The radially boosted observer will find that the volume that lies in front of him is Lorentz-contracted in the radial direction. The photons coming from the front side therefore come from a radially "squeezed" volume. But any volume contraction in the radial direction will reduce the energy of the photons because of the negative radial pressure. The blue-shift of the photons due to the (kinematical) Doppler-shift will be exactly compensated by the red-shift originating from the pressure-induced Lorentz contraction. A similar effect occurs for the photons coming from the rear, i.e. moving in the same direction as the observer.

Although perhaps some new insights are required in order to resolve the problem of relating the observations in the different frames in a satisfactory

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<sup>46</sup>This result could also have been obtained by calculating the pressure-induced energy-change in the co-moving frame. Quite interestingly this change is zero for photons, because the proper radial extension of a geodesically moving shell of photons remains constant in the co-moving frame. Therefore the energy-density of the photons in the co-moving frame evolves inverse proportional to the proper surface area of the shell, i.e.  $\bar{\rho}_\gamma \propto 1/r^2$ . Assuming that the photon number in the shell remains constant in the co-moving frame (which is equivalent to assuming a thermal spectrum) we then find  $\bar{n}_\gamma \propto 1/r^{3/2}$ .

fashion, nature appears to have taken a definite point of view: If we live in a large holostar, we clearly are in the position of the co-moving observer. Except for a small dipole anisotropy, which can be explained by the small relative motion of our local group with respect to the isotropic Hubble-flow, the CMBR is isotropic. Furthermore, in Planck-units the CMBR-temperature is  $T_{CMBR} \simeq 2 \cdot 10^{-32} T_{Pl}$ , whereas the radius of the observable universe is roughly  $r \simeq 9 \cdot 10^{60} r_{Pl}$  and the mass-density  $\rho \simeq 5 \cdot 10^{-124} \rho_{Pl}$ . These simple figures strongly suggest, that the  $T \propto 1/\sqrt{r}$  and  $\rho \propto T^4$  laws are realized in our universe in the system of the co-moving observer.

### 2.23 On the baryon to photon ratio and nucleosynthesis

The discussion of the previous section has paved the way to address the problem of nucleosynthesis in the holostar universe. A quite remarkable by-product of the discussion in this section is a surprisingly simple explanation for the baryon to photon ratio in the universe.

We have seen in the previous section that the number density of massive particles  $\overline{n}_m$  and photons  $\overline{n}_\gamma$  in the co-moving frame develop as:

$$\overline{n}_m \propto \frac{1}{r^2} \propto T^4 \quad (138)$$

and

$$\overline{n}_\gamma \propto \frac{1}{r^{\frac{3}{2}}} \propto T^3 \quad (139)$$

As consequence the ratio of the energy-densities per proper volume of massive particles to photons remains constant in the co-moving frame in the holostar universe<sup>47</sup>, i.e:

$$\overline{\rho}_m \propto \overline{\rho}_\gamma \propto \frac{1}{r^2} \propto T^4 \quad (140)$$

Comparing the energy density of electrons  $\overline{\rho}_e$  and photons  $\overline{\rho}_\gamma$  at the present time, we find that they are almost equal. In fact, assuming the chemical potential of the photons to be zero, the respective energy densities turn out as

$$\overline{\rho}_\gamma = 8.99 \cdot 10^{-128} \rho_{Pl} \quad (141)$$

and

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<sup>47</sup>Note that  $\rho_\gamma \propto \rho_m \propto 1/r^2$  has already been established in the coordinate frame.

$$\overline{\rho_e} = 2.23 \cdot 10^{-127} \rho_{Pl} = 2.52 \overline{\rho_\gamma} \quad (142)$$

if we assume an electrically uncharged universe, a proton to nucleon ratio of 7/8 and a universe consisting predominantly out of nucleons (no significant dark matter component).  $\rho_{Pl}$  is the Planck-density,  $m_{Pl}/r_{Pl}^3$ .

In the context of the standard cosmological model this fact appears as a very lucky coincidence, which happens just at the current age of the universe and won't last long: The energy density of radiation and matter evolve differently in the standard cosmological model. Basically  $\rho_m \propto T^3$ , whereas  $\rho_\gamma \propto T^4$ , so that the energy density of the radiation falls off with  $T$  compared to the energy density of the massive particles.<sup>48</sup>

The particular value of the baryon to photon ratio  $\eta$  is a free parameter in the standard cosmological model. The most recent experimental determination of  $\eta$  via primordial nucleosynthesis is given by [37] as  $\eta \approx 6.5 \cdot 10^{-10}$  (at the time of nucleosynthesis). Unfortunately we still lack an established theoretical framework by which this value could be calculated or even roughly estimated from first principles. In the holostar the value of  $\eta$  is linked to the nearly constant energy densities of the different fundamental particle species. Whenever we know what the ratio of the energy densities should be, we can estimate the present value of  $\eta$ .

Can the ratio between the energy densities of photons and electrons in the holostar universe be predicted by first principles? In order to answer this question let us turn our clocks backward until the radiation temperature in the holostar registers somewhere above the electron rest mass, but well below the mass of the muon or pi-meson (for example  $T \approx 1 - 10 \text{ MeV}$ ). The rather fast electromagnetic reactions at the high temperatures and densities ensure that the energy is distributed nearly equally among the relativistic degrees of freedom of electrons/positrons and photons, respectively.<sup>49</sup> When the temperature

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<sup>48</sup>The standard cosmological model assumes that the number ratio of baryons to photons  $\eta$  remains constant in the expanding universe. With the recent WMAP data this ratio is now estimated (at the time of nucleosynthesis) as  $\eta \simeq 6.5 \cdot 10^{-10}$ . The postulate  $\eta = \text{const}$  is quite different to the evolution of the different particle species in the holostar, where rather the energy densities, and not the number-densities of the fundamental particle species remain constant during the evolution.

<sup>49</sup>I never found the basic assumption of the standard cosmological model utterly convincing, according to which the photon to baryon ratio should be frozen at a constant value that is postulated to have remained nearly constant from the time of baryogenesis ( $T \approx 1 \text{ GeV}$ ) to the time of nucleosynthesis ( $T \approx 0.1 \text{ MeV}$ ) and forever thereon. There is no problem with such an assumption after the time where photons and matter became chemically decoupled, i.e. after nucleosynthesis ended. It is not difficult to verify that in a homogeneously expanding universe with negligible pressure and negligible particle-changing interactions (i.e. no particle creation/destruction) the particle numbers of the different stable species in any volume co-moving with the expansion is conserved. Note, however, that this feature stands and falls with the assumption of a homogenous universe with a universal cosmological time, i.e. a universe which looks exactly the same at any spatial position at any fixed value of the cosmological time. In such a universe the particles (massive or mass-less alike) have no other way to "go"

falls below the threshold defined by the electron-mass, the last (few!) remaining positrons are quickly dispatched, so that the number of photons should be nearly equal to the number of the relic electrons. The same will be true for the respective energy densities, so that as a very rough estimate we can assume, that the energy densities of photons and electrons are equal at decoupling.<sup>50</sup>

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At temperatures below nucleosynthesis, i.e.  $T < 0.1\text{MeV}$ , the energy density than to move with the geodesically expanding volume. In the holostar universe the situation is different, as the particles move through a static space-time and can "go anywhere they want", as long as they obey the local equations of motion (which are not equivalent with the geodesic equations of motion!). But even in a homogeneously expanding universe it requires quite a bit of "fine-tuning" with respect to the physics if the photon to electron ratio should be vastly different from unity at or slightly below the temperature where photons and electrons chemically decouple, i.e. at  $T < m_e/3 \approx 0.15\text{MeV}$ . Taking into account the strength of the electro-weak and the strong interactions in the energy range from  $0.2 - 1000\text{MeV}$  and taking into account the expansion rate of the universe during this period, the mutual interactions are fast enough to maintain a thermal spectrum in the whole range  $T \approx 0.2 \dots 1000\text{MeV}$  in the standard cosmological model. But in near thermal equilibrium the energy density of the ultra-relativistic electrons (and positrons) must be comparable to that of the photons. This conclusion remains true even if there is an appreciable net baryon lepton asymmetry and is independent from the ratio of photons to baryons that might have been produced by the mutual annihilation of baryon/anti-baryon pairs at the time of baryogenesis: In the cosmological time period between baryogenesis and nucleosynthesis, when the temperature falls from  $1\text{GeV}$  to  $0.2\text{MeV}$ , the Hubble-expansion is not fast enough to shut off the electro-magnetic interactions. Therefore, when the temperature has reached the electron mass threshold, the energy density of photons and electrons will have become comparable to each other, equal within one order of magnitude if the possibly non-zero chemical potentials of the particles are taken into account. In fact, the same argument applies to baryogenesis. It is not understandable that an asymmetry on the order of  $10^9$  photons to baryons could have arisen in the first place: At the time where the annihilation of baryons/antibaryons starts to set in, the energy densities of baryons and photons are nearly equal. Even if there were a ratio of  $10^9 + 1$  baryons to  $10^9$  anti-baryons at  $T \approx 1\text{GeV}/2.7$ , the annihilation of baryon/anti-baryon pairs wouldn't happen instantaneously. It rather has the effect that energy is transferred to the photons (and the other relativistic particles), keeping the temperature constant. Yet the universe doesn't halt its expansion. Rather the continued expansion against the (positive) radiation pressure will gradually reduce the energy density of all particle species at a nearly constant temperature. There is no conservation law for photons which would forbid photons to be destroyed in favor of maintaining a constant energy/temperature. Furthermore, at the nearly constant temperature of the phase transition the photons are still in close thermal contact with the baryons, so that the photon and baryon energy densities remain nearly equal. As the temperature remains constant (with the mean momentum of the photons equalling the baryon rest mass), the number-densities of photons and baryons (and of the yet highly relativistic electrons, muons and neutrinos, Pi-mesons etc.) will remain nearly equal as well. The process stops, when a baryon density slightly above the final density of the relic baryons is reached, for example two baryons to one anti-baryon. Only then will the temperature drop below the threshold for the creation of baryon/anti-baryon pairs, effectively shutting off the reactions that maintained thermal equilibrium up to this point. A similar sequence of events happens at the electron-mass threshold, so that it is quite inconceivable that we should encounter the vast number of  $10^9$  photons per relic electron (or baryon) just below the electron mass threshold. Only after photons and electrons have decoupled chemically, i.e. well below the electron mass threshold, the photon to electron ratio in an expanding universe develops independently from a thermal distribution.

<sup>50</sup>In the simple analysis here I neglect the difference in energy-densities of photons and electrons due to the fact, that photons are bosons whereas electrons are fermions. Likewise the non-negligible effects of any non-zero chemical potentials of photons and electrons are neglected.

sities of photons and electrons in the holostar evolve nearly independently. Even when the reactions that have maintained thermal equilibrium before have ceased, the energy-densities of the two particles species will be maintained at a nearly constant ratio, whose exact value is determined by the physics at the time-period where electrons and photons have decoupled chemically.<sup>51</sup> An exact determination of this ratio is beyond the scope of this paper. Quite likely chemical potentials will play an important role (see [30]). However, it is quite encouraging that the very rough estimate  $\rho_e \approx \rho_\gamma$  at the time of chemical decoupling is within a factor of three of the experimental value  $\rho_e \approx 2.5\rho_\gamma$  determined at the present time.

The discussion above allows us to make a rough estimate for the value of  $\eta$  at the present time. Assuming that photons and electrons decoupled chemically at  $T = m_e/3$ , assuming equal energy-densities of electrons and photons at this time we find:

$$\eta_{today} \approx \frac{3T_{CMBR}}{m_e} = 1.38 \cdot 10^{-9} \quad (143)$$

Remarkably, this very rough estimate is only a factor of 2 higher than the WMAP-result  $\eta_{WMAP} \approx 6.5 \cdot 10^{-10}$ .

We can also compare the value in equation (143) to the maximum possible value of the baryon to photon ratio in the universe today. Under the assumption that there is no significant dark matter-component  $\eta_{max}$  can be estimated from the cosmological number densities of photons and electrons.<sup>52</sup> We find  $\eta_{max} \approx 3.14 \cdot 10^{-9}$ , which is a factor of two higher than the estimate of equation (143).

We are now in the position to discuss nucleosynthesis in the holostar universe. The primordial synthesis of the light elements proceeds somewhat differently as in the standard cosmological model. The two key parameters governing the respective reaction rates, the number density of the nucleons (baryons)  $n_b$  at the temperature of nucleosynthesis and the (Hubble) expansion rate, turn out to be significantly different in both models.

In the standard cosmological model the number-density of the nucleons depends on the cube of the temperature, i.e.  $n_b \propto 1/R^3 \propto T^3$ , whereas according to the discussion above the co-moving observer in the holostar finds that the

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<sup>51</sup>It is quite remarkable, that the energy-densities of the different fundamental particle species in the holostar evolve with a constant ratio throughout the whole holostar: At high temperature the nearly equal energy densities are maintained by the reactions establishing thermal equilibrium. At low temperatures the geodesic motion in combination with the negative radial pressure ensures, that the massive and massless particles maintain a constant energy ratio during their motion.

<sup>52</sup>The photon number density is determined by the Planck formula, assuming the chemical potential of the photons to be zero. The maximum value for the electron number density is determined from the total matter-density of the universe according to WMAP, assuming no significant dark matter component, a proton to nucleon ratio of 7/8 and an uncharged universe with no significant antimatter.

number density of non-relativistic particles scales as the fourth power of the temperature,  $\overline{n_b} \propto 1/r^2 \propto T^4$ . Therefore the number-density of the nucleons at the temperature of nucleosynthesis will be roughly nine orders of magnitude higher than in the standard model.

This won't have a large effect on the ratio of primordial Helium-4 to Hydrogen. The numerical value of the He/H ratio is mainly due to the n/p ratio in thermal equilibrium at  $T \approx 0.8 \text{ MeV}$ , i.e. the temperature where the weak reactions are shut off.<sup>53</sup> However, the higher number-density will have a considerable effect on the amount of Deuterium, Helium-3 and Lithium-7 produced in the first few seconds of the universe.

The relative abundance of these elements depends quite sensitively on the nuclear reaction rates, which are proportional to the number-densities of the reacting species. In order to accurately calculate the relative number densities of all the other elements, excluding H and He-4, one has to consider the dynamic process in which the nuclear reactions compete against the cosmic expansion. Eventually the nuclear reaction rates will fall below the expansion rate, ending the period of nucleosynthesis. Therefore the second key parameter in primordial nucleosynthesis, besides the nucleon number-densities, is the cosmic expansion rate, which is proportional to the Hubble-value.

In the matter dominated era the standard model predicts  $H \propto 1/t \propto 1/R^{3/2} \propto T^{3/2}$ , whereas in the radiation dominated era  $H \propto T^2$ . In the radiation dominated era the dependence of the expansion rate on the temperature is equal in both models. Taking the different dependencies in the matter-dominated era into account, one can relate the Hubble-constant  $H_h$  in the holostar universe and the Hubble-constant  $H_s$  in the standard model at the time of nucleosynthesis. We find:

$$H_h \approx \sqrt{\frac{T_{eq}}{T_{CMBR}}} H_s = \sqrt{z_{eq}} H_s \simeq 60 H_s \quad (144)$$

where  $z_{eq} \approx 3450$  is the red-shift at which the standard model becomes radiation dominated, according to the recent WMAP findings. The above result is quite consistent with the fact, that the "age" of the universe at  $T = 0.1 \text{ MeV}$  is roughly  $\tau \approx 7s$  in the holostar, whereas in the standard cosmological model we find  $t \approx 200s$  at the same temperature.

It should be evident, that due to the differences in the two key parameters nucleosynthesis in the holostar universe will proceed quite differently from the standard model. Without actually solving the rate equations it is difficult to

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<sup>53</sup>Neutron decay during the time where the temperature drops to roughly  $0.1 \text{ MeV}$ , i.e. the temperature where deuterons are produced in significant numbers (and quickly end up in the more stable  $He^4$ ), also plays a non-negligible role in the standard cosmological model. Note that the shut-off temperature of the weak interactions, which is  $0.8 \text{ MeV}$  in the standard model, depends on the Hubble rate, and thus will be (slightly) different in the holostar.

make quantitative predictions. In general the higher number density of nucleons in the holostar will lead to a more effective conversion of Deuterons to the stable Helium-4 nucleus, reducing the relative abundance of the few Deuterium nuclei that survive their conversion to He-4. On the other hand, the significantly larger expansion rate enhances the chance, that the less stable nuclei such as Deuterium and He-3 will survive through the much shorter time period of primordial nucleosynthesis. The nuclear reactions are shut-off faster, providing some amount of "compensation" for the faster reaction rates due to the higher number densities. Yet it would be quite a coincidence, if nucleosynthesis in the holostar would lead to the same abundances of Deuterium, Helium-3 and Lithium-7 as in the standard cosmological model. Whether the holostar model of the universe can explain the experimentally determined abundances of the light elements in a satisfactory fashion, therefore must be considered as an open question.

## 2.24 The Unruh temperature

A massive particle at rest in the  $(r, \theta, \varphi, t)$  coordinate system is subject to a geodesic acceleration given by equation (55). With this acceleration the following Unruh-temperature can be associated

$$T_U = \frac{g\hbar}{2\pi} = \frac{\hbar}{4\pi r} \sqrt{\frac{r_0}{r}} = T_\gamma \frac{r_0}{r} \quad (145)$$

where  $T_\gamma$  is the local radiation temperature given by equation (51).

At  $r \approx r_0$  the Unruh temperature is comparable to the Planck-temperature. Therefore particles with masses up to the Planck-mass can be created out of vacuum in the vicinity of a roughly Planck-size region close to the center of the holostar.

If we multiply the Unruh-temperature with the local radiation temperature of equation (51) we find the following interesting relation, which doesn't include the - unknown - parameter  $r_0$ :

$$T_U T_\gamma = \frac{\hbar^2}{16\pi^2 r^2} = \frac{\hbar^2}{2\pi} \rho \quad (146)$$

The Unruh temperature, as derived in equation (145), only applies to an observer at rest in the holostar. It cannot be experienced by an observer moving geodesically, because such an observer is by definition unaccelerated in his own frame of reference.

However, pure geodesic motion is not possible within a holostar, due to the pressure. Even in the low-density regions of a holostar where the motion is

predominantly geodesic, the pressure will provide a small deceleration, which can be estimated by boosting the pressure induced acceleration,  $a_P$ , given by

$$a_P = \frac{\sigma}{m} P = \frac{\sigma}{m} \frac{1}{8\pi r^2} \quad (147)$$

to the frame of the co-moving observer. If we denote the frame of the co-moving observer with an overline, we get  $\overline{a_P} = \gamma^3 a_P$ , because the deceleration  $a_P$  is parallel to the (radial) boost.  $\gamma$  is given by equation (73). Therefore the acceleration (or rather deceleration) due to the pressure in the frame of the co-moving observer is given by:

$$\overline{a_P} = \frac{\sigma}{m} \frac{1}{8\pi r^2} \left( \frac{r}{r_i} \right)^{\frac{3}{2}} = \frac{\sigma}{2m} \frac{1}{4\pi \sqrt{r r_i^3}} \quad (148)$$

This acceleration can be associated with an Unruh temperature, which the co-moving observer should be able to measure in principle. It is quite remarkable, that the Unruh-temperature in the frame of the co-moving observer turns out to be exactly proportional to the local radiation temperature  $T_\gamma$ :

$$\overline{T_U}(r) = \frac{\hbar \overline{a_P}}{2\pi} = \frac{\sigma}{4\pi m r_i} \sqrt{\frac{r_0}{r_i}} T_\gamma(r) \quad (149)$$

For  $r_i \gg r_0$  the Unruh-temperature will be much lower than the radiation temperature. Whenever the particles move geodesically, the ratio of their Unruh-temperature to the local radiation temperature is nearly constant and given by:

$$\frac{\overline{T_U}}{T_\gamma} = \frac{\sigma}{\hbar} \frac{T_i}{m} \frac{r_0}{r_i} \quad (150)$$

$T_i = T_\gamma(r_i)$  is the local radiation temperature at the turning point of the motion of the geodesically moving particle (or rather the temperature at the radial coordinate position  $r_i$ , where the motion has become geodesical). According to the discussion in 2.11 and the results obtained in [28]  $\sigma/\hbar \approx s = m/T_i$ , therefore the factor in front of  $r_0/r_i$  in equation (150) is quite close if not equal to one, so that

$$\frac{\overline{T_U}}{T_\gamma} \simeq \frac{r_0}{r_i} \quad (151)$$

For a particle of nearly Planck-mass emitted from the central region of the holostar, the Unruh-temperature will be comparable to the radiation temperature, if the particle can be considered to move geodesically from the beginning.

This opens up the possibility to explain the isotropy of the radiation temperature in the co-moving frame by the Unruh-effect, since the Unruh-temperature is always isotropic in the accelerated frame. In sections 2.18 and 2.20 it has been shown, that the Hubble value and the evolution of the density fluctuations in a holostar-universe are compatible with the experimentally determined values of our universe, if  $\tilde{r}_i/r_0 = 3/4$ , where  $\tilde{r}_i$  is the "fictitious" zero-velocity turning point of the motion. In fact, for more realistic scenarios (non-instantaneous decoupling) we find  $r_i \simeq r_0$ . Note also, that in section 2.19 it has been shown that the true turning point of the motion  $r_i$  (for  $\tilde{r}_i/r_0 = 3/4$ ) should lie roughly at  $r_i \approx 1.08 r_0$  (equation (108)). This strongly suggests, that the microwave-background temperature is in fact equal to the Unruh temperature, i.e.  $r_i = r_0$ , so that  $T_\gamma = \overline{T}_U$  and thus  $H = 1/r = 1/\tau$ .

However, in order to attain an Unruh-temperature comparable to the temperature of the CMBR, the proper acceleration in the co-moving frame would have to be enormous:  $a = 2\pi\overline{T}_U/\hbar \approx 10^{20} m/s^2$  for  $\overline{T}_U \simeq 2.73K$ . If the microwave background radiation is supposed to be nothing else than Unruh radiation, why then don't we notice this enormous acceleration? This answer to this open question might be found in the pressure<sup>54</sup> or - possibly - in the rotation of the holostar.

### 3 The holostar as a unified description for the fundamental building blocks of nature?

As has been demonstrated in the previous chapter, the holostar has properties which are in many respects similar to the properties of black holes and the universe.

It is quite remarkable, that the fairly simple model of the holostar appears to have the potential to explain the properties of objects, that so far have been treated as distinct building blocks of nature, in one uniform description. Black holes and the universe didn't appear to have much in common, although both are strongly gravitating systems. The holostar solution points at a deeper connection between self gravitating systems of any size. The holostar solution might even be of some relevance for elementary particles.

This chapter is dedicated to a first preliminary discussion of the question, whether the holostar can serve as an alternative - possibly unified - description for black holes, the universe and elementary particles.

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<sup>54</sup>A diver in water can "compensate" the (constant) gravitational field of the earth if his mean density is equal to the density of the pressurized fluid (water) in which he floats. Maybe the negative pressure, which has the same energy density as the "ordinary matter", provides a similar compensation. This is even more suggested by the fact, that in general relativity the "true" coordinate independent gravitational effect is not a constant acceleration field (which can be transformed away), but rather the tidal acceleration, which is quite low in the outer regions of the holostar.

### 3.1 The holostar as alternative to black holes?

The holostar possesses properties very similar to properties attributed to a black hole. It has an entropy and temperature proportional, if not equal to the Hawking entropy and temperature. Its exterior space-time metric is equal to that of a black hole, disregarding the Planck coordinate distance between the membrane and the gravitational radius. Therefore, as viewed from an exterior observer, the holostar is indistinguishable from a black hole.

With respect to the interior space-time, i.e. the space-time within the event horizon (or within the membrane), there are profound differences:

A black hole has no interior matter, except - presumably - a point mass  $M$  at the position of its central singularity.<sup>55</sup> The number and the nature of the interior particles of a black hole, or rather the particles that have gone into the black hole, is undefined. Any concentric sphere within the event horizon is a trapped surface. The holostar has a continuous interior matter distribution, no singularity at its center, no trapped surfaces and - as will be shown in [30] - a definite number of interior particles proportional to its boundary area. The absence of trapped surfaces and of an accompanying singularity is a desirable feature in its own right. Furthermore, the holostar appears to be the most radical fulfillment of the holographic principle: The number of the holostar's interior particles is exactly proportional to its proper surface area. Many researchers believe the holographic principle to be one of the basic ingredients from which a possible future universal theory of quantum gravitation can be formed.

A black hole has an event horizon. The unique properties of the event horizon, i.e. its constant surface gravity and its (classically) non-decreasing area, determine the thermodynamic properties of a black hole, i.e. its Hawking temperature and entropy. The holostar's thermodynamic properties are determined by its interior matter configuration. Therefore the holostar solution "needs" no event horizon. In fact, it possesses no event horizon, because the metric coefficients  $B$  and  $1/A$  never become zero in the whole space-time. Note, however, that for a large holostar the minimum value of  $B$  becomes arbitrarily close to zero.  $B$  reaches its lowest value at the boundary  $r_h = r_+ + r_0$ :

$$B_{min} = B(r_h) = \frac{r_0}{r_h} = \frac{1}{1 + \frac{r_+}{r_0}} \quad (152)$$

For a holostar with a gravitational radius of  $n$  Planck lengths, i.e.  $r_+ = nr_0$ , one gets  $B_{min} = 1/(n+1)$  and  $A_{max} = n+1$ . A holostar with the mass of the sun has  $n \approx 2 \cdot 10^{38}$  and therefore  $B_{min} \approx 10^{-38}$ .

Instead of an event horizon the holostar possesses a two dimensional membrane of tangential pressure, whose properties are very similar to the properties

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<sup>55</sup>A rotating Kerr-black hole has a ring-singularity.

of the event horizon of a black hole as expressed by the membrane paradigm [31, 40].

The absence of an event horizon is a desirable feature of the holostar. If there is no event horizon, there is no information paradox. Unitary evolution of states is possible throughout the whole space-time.

From the viewpoint of an exterior observer the holostar appears as a viable alternative to a black hole. It is in most respects similar to a black hole but doesn't appear to be plagued with the undesirable consequences of an event horizon (information paradox) and of a central singularity (breakdown of causality and/or predictability).

Furthermore the holostar's interior is truly Machian. What matters are the relative positions and motions of the interior particles with respect to the whole object. The holostar's spherical membrane serves as the common reference, not the ill-defined notion of empty asymptotic Minkowski space.

Both the holostar and a black hole are genuine solutions to the field equation of general relativity. From the viewpoint of the theory both solutions have a good chance to be physically realized in the real world. At the time this paper is written the holostar solution looks like an attractive alternative to the black hole solutions. However, only future research - theoretical and experimental - will be able to answer the question, what alternative, if any, will provide a better description of the phenomena of the real world. Some more evidence in favor of the holostar solution will be presented in [30, 28].

### 3.2 The holostar as a self-consistent model for the universe?

A large holostar has some of the properties, which can be found in the universe in its actual state today: At any interior position there is an isotropic radiation background with a definite temperature proportional to  $1/\sqrt{r}$ . Geodesic motion of photons preserves the Planck-distribution. Massive particles acquire a radially directed outward motion, during which the matter-density decreases over proper time. Likewise the temperature of the background radiation decreases. The temperature and mass density within the holostar are related by the following formula,  $\rho = 2^5 \pi^3 r_0^2 T^4$ , which yields results consistent with the observations, when the mass density and microwave-background temperature of the universe are used as input and  $r_0$  is set to roughly twice the Planck-length.

The theoretical description of the universe's evolution is in many aspects similar to the standard cosmological model. In the standard model the energy density is related to the cosmological time as  $\rho \propto 1/t^2$ . This relation is valid in the matter dominated as well as in the radiation dominated epoch. It is also valid in the holostar universe, if  $t$  is interpreted as the proper time of the co-moving observer. The standard cosmological model furthermore has

the following dependence between the total energy density and the radiation temperature:  $\rho \propto T^4$ . In the holostar universe the same relation is valid.

On the other hand, the holostar has some properties, which might not be compatible with our universe: When massive particles and photons are completely decoupled, the ratio of the number-density of photons with respect to the number density of massive particles is predicted to increase over time in the holostar-universe ( $n_m \propto 1/r^2$ , whereas  $n_\gamma \propto 1/r^{3/2}$ ). The standard cosmological models assumes that this ratio remains nearly constant up to high redshift. Bigbang nucleosynthesis in the standard cosmological models wouldn't be compatible with the observations, if the photon to baryon ratio at high redshift would be very different from today.

Yet the holostar appears to have the potential to explain some phenomena, which are unexplained in the standard cosmological models:

The standard model has the  $\Omega$ - or flatness-problem. Why is  $\Omega$  today so close to 1? If  $\Omega$  is not exactly 1, it must have been close to 1 in the Planck-era to an accuracy of better than  $10^{-60}$ , i.e. the ratio of one Planck length to the radius of the observable universe today. Such a finetuning is highly improbable. One would expect  $\Omega = 1$ , exactly. However, in the standard cosmological models  $\Omega$  is a free parameter. There is no necessity to set it equal to one. The holostar "solves" this problem in that there is no free parameter. The total energy density is completely fixed. Any other total energy density would result in a very different metric, for which most of the results presented in this paper, as well as in [30, 28], would not hold.

The standard model has the cosmological constant problem: The recent supernova-experiments [27, 32, 26] indicate that the universe is accelerating. In the standard cosmological models such an acceleration is "explained" by a positive cosmological constant,  $\Lambda > 0$ . However, the natural value of the cosmological constant would be equal to the Planck-density, whereas the cosmological constant today is roughly a factor of  $10^{124}$  smaller than its "natural" value. Such a huge discrepancy suggests, that the cosmological constant should be exactly zero. The supernova red-shift surveys, however, demand a cosmological constant which amounts to roughly 0.7 of the critical mass-density today. Why does  $\Lambda$  have this particular value? Furthermore,  $\Lambda$  and the mass-density scale differently with  $\tau$  (or  $r$ ). Why then do we just happen to live in an epoch where both values are so close?<sup>56</sup> The holostar solution solves the problem of an accelerating universe without need for a cosmological constant. The holostar solution in fact requires a cosmological constant which is exactly zero, which in consequence can be interpreted as a strong indication that supersymmetry might well be an exact symmetry of nature.

The standard model has the so called horizon problem. This problem can be traced to the fact, that in the standard cosmological models the scale factor of

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<sup>56</sup>Some cosmological models therefore assume a time-varying cosmological constant, which however is quite difficult to incorporate into the the context of general relativity.

the universe varies as  $r \propto t^{2/3}$  in the matter-dominated era and as  $r \propto t^{1/2}$  in the radiation dominated era, whereas the Hubble-length varies proportional to  $r_H \propto t$ . If the scale factor varies slower than the Hubble length, the region that can be seen by an observer today will have originated from causally unconnected regions in the past. For example, at red-shift  $z_{eq} \approx 1000$ , i.e. when matter and radiation have decoupled according to the standard model, the radius of the observable universe consisted of roughly  $d_e/d_c \approx \sqrt{z_{eq}} \approx 30$  causally disconnected radial patches, or  $30^3$  regions.  $d_e$  is the distance scale of the expansion,  $d_c$  of the causally connected regions. The horizon problem becomes even worse in the radiation dominated era.<sup>57</sup> The horizon problem makes it difficult to explain why the CMBR is isotropic to such high degree. Inflation is a possible solution to this problem. However, it is far from clear how and why inflation started or ended. Furthermore, the inflationary scenarios need quite a bit of finetuning and there appears to be no theoretical framework that could effectively restrain the different scenarios and/or their parameters. The holostar space-time solves the horizon problem in a quite elegant way. The Hubble length  $r$  is always proportional to the scale-factor in the frame of the co-moving observer, as  $r \propto \tau$  and  $H \propto 1/r$ , therefore everything that is visible to the co-moving observer today, was causally connected to him in the past.

Related to the horizon problem is the problem of the scale factor. With the  $T \propto 1/r$ -law, in the standard cosmological models the scale factor is  $r \approx 10^{30} r_{Pl}$  at the Planck-temperature. Why would nature choose just this number? Inflation can solve this problem, albeit not in a truly convincing way. The holostar universe with  $T \propto 1/\sqrt{r}$  and  $r \propto \tau$  elegantly gets rid of this problem. The scale factor evolves proportional (and within the errors equal!) to the Hubble-length. At the Planck-temperature, both the scale-factor and the Hubble-length are nearly equal to the Planck-length.

Inflation was originally introduced in order to explain the so called monopole-problem. The standard cosmological model without inflation predicts far too many monopoles. Inflation, if it sets in at the right time, thins out the number of monopoles to a number consistent with the observations. It might be, that the holostar universe has an alternative solution to the problem of monopoles or other heavy particles: Heavy particles will acquire geodesic motion very early in the holostar, i.e. at small  $r$ -coordinate value. But pure geodesic motion in the holostar universe tends to thin out the non-relativistic particles with respect to the still relativistic particles.

If we actually live in a large holostar, we should be able to determine our current radial position  $r$  by the formulae given in the previous sections. In principal it safest to determine  $r$  by the total matter-density via  $\rho = 1/(8\pi r^2)$ , as no unknown parameters enter into this determination. Using the recent WMAP-data we find:

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<sup>57</sup>In the radiation dominated era one finds  $d_e/d_c \approx z$ .

$$r = \frac{1}{\sqrt{8\pi\rho}} = 9.129 \cdot 10^{60} r_{Pl} = 1.560 \cdot 10^{10} ly$$

This is quite close to the value  $r \approx 1.5 \cdot 10^{10} ly \approx 8.8 \cdot 10^{60} r_{Pl}$ , which was thought to be the radius of the observable universe for a rather long period of time.

However, the (total) matter density is difficult to determine experimentally. Although other estimates of the total mass-energy density, for example by examining the rotation curves of galaxies and clusters, all lie within the same range, the accuracy of the measurements (including systematic errors) is most likely not better than 5 %.

A much better accuracy can be achieved through the precise measurements of the microwave background radiation, whose temperature is known to roughly 0.1%. The determination of  $r$  by the *CMBR*-temperature, however, requires the measurement or theoretical determination of the "fundamental area"  $r_0^2$ . Its value can be determined experimentally from both the matter-density and the radiation temperature, or theoretically as suggested in [28]. If we use the theoretical value ( $r_0^2 \simeq 2\sqrt{3}\hbar$ ) at low energies we find:

$$r = \frac{\hbar^2}{16\pi^2 T^2 r_0} = 9.180 \cdot 10^{60} r_{Pl} = 1.569 \cdot 10^{10} ly$$

In a large holostar the momentary value of the  $r$ -coordinate can also be calculated from the local Hubble-value. In order to do this, the starting point of the motion  $r_i$  and the fundamental length parameter  $r_0$ , or rather their ratio, have to be known. Theoretically one would expect  $r_i \simeq r_0$  (see for example section 2.24), whereas experiments point to a somewhat lower value. If we use  $r_i/r_0 \approx 1$  and  $H = 71(km/s)/Mpc$  we find:

$$r = \sqrt{\frac{r_0}{r_i}} \frac{1}{H} = 8.062 \cdot 10^{60} r_{Pl} = 1.378 \cdot 10^{10} ly$$

Very remarkably, all three different methods for the determination of  $r$  give quite consistent results, which agree by 15%. A rather large deviation comes from the Hubble-value, which is not quite unexpected taking into account the difficulty of matching the different cosmological length scales. Note however, that the rather good agreement depends on the assumption  $r_0 \simeq 1.87 r_{Pl}$  and  $r_i/r_0 \simeq 1$ . These assumptions, especially the second, could well prove wrong by a substantial factor.

From a theoretical point of view  $r_i = r_0$  is interesting because it allows us to interpret the microwave background radiation as Unruh radiation. If this turns out to be the right ansatz, the Hubble constant can be predicted. Its value should be:

$$H = 62.36 \frac{km/s}{Mpc} \quad (153)$$

This value is well within the measurement errors, which are mostly due to the calibration of the "intermediate" ladder of the cosmological distance scale via the Cepheid variables.

Without strong theoretical support for a definite value of  $r_0$  the matter-density seems to be the best way to determine  $r$ . If the fundamental length parameter  $r_0$  can be pinned down theoretically, such as suggested in [28], the extremely precise measurements of the microwave background temperature will give the best estimate for  $r$ .

The simplicity of the holostar solution and the fact that it has no adjustable parameters<sup>58</sup> makes the holostar solution quite attractive as an alternate model for the universe, a model that can be quite easily verified or falsified.

At the time this paper is written it not clear, whether the holostar will provide a serious alternative to the standard cosmological model. It has the potential to solve many of the known problems of the standard cosmological model. But the standard cosmological model has been very successful so far. One of its great achievements is the remarkably accurate explanation for the distribution of the light elements, produced by bigbang nucleosynthesis. Although the holostar universe is in many respects similar to the standard cosmological models, it is doubtful that the synthesis of the light elements will proceed in exactly the same way as in the standard model. It would be quite a coincidence if the holostar could give a similarly accurate agreement between theory and observation.<sup>59</sup>

Furthermore, it is not altogether clear how the "true" motion of the massive particles within the cosmic fluid will turn out. Therefore some results from

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<sup>58</sup>Possibly an overall charge  $Q$  or angular momentum could be included.  $r_0$  is not regarded as an adjustable parameter: The analysis here, as well as in [30, 28] strongly suggest,  $r_0^2 \simeq 4\sqrt{3/4}r_{Pl}$  at low energies. As long as the theoretical "prediction"  $r_i = r_0$  hasn't been confirmed independently  $r_i$  should be regarded as an adjustable parameter. It is encouraging, however, that  $r_i/r_0$ , as determined from the relative growth of the density perturbations after radiation and matter became kinematically decoupled, is quite close to the value determined from the measurements of the Hubble-constant.

<sup>59</sup>Note, that according to [30] chemical potentials and supersymmetry appear to play an important role in the holostar. Supersymmetry within the holostar requires that the ratio of the energy-densities of fermions to bosons isn't given by the usual factor 7/8, but rather each fermionic degree of freedom has an energy density which is higher than that of a bosonic degree of freedom by roughly a factor of 12.5. If the energy density in the radiation (neutrinos, electrons and photons) at the time of nucleosynthesis is normalized to the photon energy density, assuming a factor of 7/8 for the electrons and neutrinos, one underestimates the total energy density by a factor of roughly 2.9 (if the three neutrinos come only in one helicity-state) or 4.6 (three neutrinos with 2 helicity states). If the energy-density is underestimated, the expansion rate comes out too low. The "true" expansion rate at the time of nucleosynthesis will have been higher. A higher expansion rate, however, requires a higher number density of the baryons, in order to get the same relative abundances of the light elements. This might partly explain the missing factor of 6, assuming  $\Omega_m \approx 0.26$  and  $\Omega_b \approx 0.04$

the simple analysis in this paper might have to be revised. There also is the problem of relating the observations in the frames of the co-moving observer and the observer at rest, which was discussed briefly in section 2.22, but which quite certainly requires further more sophisticated consideration.

On the other hand, it is difficult to believe that the remarkably consistent determination of  $r$  by three independent methods is just a numerical coincidence with no deeper physical meaning.

Therefore the question whether the holostar can serve as an alternative description of the universe should be regarded as open, hopefully decidable by research in the near future. Even if the holostar doesn't qualify as an accurate description of the universe, its particularly simple metric should make it a valuable theoretical laboratory to study the so far only poorly understood physical effects that arise in a universe with significant (anisotropic) pressure.

### 3.3 The holostar as a simple model for a self gravitating particle with zero gravitational mass ?

Another unexpected feature of the holographic solution is, that one can choose  $r_+ = 2M = 0$  and still get a genuine solution with "structure", i.e. an interior "source region" with non-zero interior matter-distribution bounded by a membrane of roughly Planck-area ( $r_+ = 0$  implies  $r_h = r_0$ ). Note that the interior "source region"  $r < r_0$  should not be considered as physically accessible for measurements. Neither should the interior matter distribution be considered as a meaningful description of the "interior structure". In fact, the holostar solution with  $r_h = r_0$  and  $M = 0$  should be regarded to have no physically relevant interior sub-structure. The only physically relevant quantity is its finite boundary area.

The  $r_+ = 0$  solution therefore might serve as a very simple, in fact the simplest possible model for a particle of Planck-size. It describes a (spin-0) particle with a gravitating mass which is classically zero, as can be seen from the properties of the metric outside the "source region" (i.e.  $A = B = 1$ ):

Although it quite unlikely that an extrapolation from the experimentally verified regime of general relativity right down to the Planck scale can be trusted quantitatively, the holographic solution hints at the possibility, that an elementary particle might be - at least approximately - describable as a self-gravitating system.

There are remarkable similarities between the properties of black holes and elementary particles. As has already been noted by Carter [7] the Kerr-Newmann solution has a gyromagnetic ratio of 2, i.e. its g-factor is equal to that of the electron (disregarding radiative corrections). The (three) quadrupole moments of the Kerr-Newman solution are  $2/3$  and  $-1/3$  in respective units, an interesting analogue to the fractional charges of the three quarks confined within the nucleon?!

It is not altogether folly to identify elementary particles with highly gravitating systems. For energies approaching the Planck-scale gravity becomes comparable to the other forces. Therefore an elementary particle will quite certainly become a strongly gravitating system at high energies. However, the identification of an elementary particle with a solution to the classical field equations of general relativity at the low end of the energy scale has so far met serious obstacles. It is difficult to explain why the masses of the light elementary particles are about 20 orders of magnitude smaller than the Planck-mass.

The common expectation, that the Planck-mass will be the minimum mass of a compact self gravitating object can be traced to the fact, that the only "natural" unit of mass which can be constructed from the three fundamental constants of nature  $c$ ,  $G$  and  $\hbar$  is the Planck mass. Thus it seems evident, that any "fundamental" theory of nature which incorporates the three constants, must have fundamental particles of roughly Planck-mass.<sup>60</sup> This quite certainly would be true, if mass always remains a "fundamental" parameter of nature, even at the most extreme energy scales.

However, there might be a slight misconception about the significance of mass in high gravitational fields. Without doubt, "mass" is a fundamental quantity both in Newton's theory of gravitation and in quantum field theories such as QED or QCD. These theories have very heavily influenced the conception of mass as a fundamental quantity of physics. In a geometric theory, such as Einstein's theory of general relativity, mass comes not as a natural property of a space-time-manifold. From a general relativistic viewpoint mass isn't a geometric property at all and enters into the equations of general relativity in a rather crude way, as stated by Einstein himself.<sup>61</sup> Furthermore, mass has dimensions of length. From the viewpoint of quantum gravity length - in contrast to area (or volume) - is a concept which is difficult to define.<sup>62</sup> Therefore it is questionable if mass will remain a fundamental parameter<sup>63</sup> in situations of high gravity, where the geometry cannot be considered static but becomes dynamic itself. In these situations it seems more "natural" to describe the interacting (geometric) objects not by their mass, but rather by the area of their boundaries. Mass appears rather as a perturbation. Surfaces as basis for the "natural" description of systems with high gravitational fields are not only motivated by recent research results, such as the holographic principle [15, 38], the study of light-sheets [5] and isolated horizons [2, 1], or M-theory, but also by the simple fact, that in natural units  $c = G = 1$  area has dimension of action

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<sup>60</sup>Supersymmetry suggests another possibility: Although the fundamental mass-scale in any supersymmetric theory is the Planck-mass, the near zero masses of the elementary particles are explained from a very precise cancellation of positive and negative contributions of bosons and fermions, due to the (hidden) supersymmetry of nature.

<sup>61</sup>Einstein once described his theory as a building, whose one side (the left, geometric side of the field equations) is "constructed of marble", whereas the right side (matter-side) is "constructed from low grade wood".

<sup>62</sup>See for example [39]

<sup>63</sup>"fundamental" is used in the current context as "optimally adapted" for the description of the phenomena.

(or angular momentum), from which we know that it is quantized in units of  $\hbar/2$ .

Therefore the "elementary" holostar with zero gravitational mass, but non-zero boundary area might serve as a very crude classical approximation for the most simple elementary particle with zero angular momentum and charge. Unfortunately none of the known particles can be identified with the "elementary" uncharged and spherically symmetric holostar-solution with  $r_h = r_0$ . Even if a spin-zero uncharged and zero mass particle exists<sup>64</sup>, it is unlikely that more than its cross-sectional area can be "predicted" by the holostar solution. Still it will be interesting to compare the properties of a charged and/or rotating holostar to the properties of the known elementary particles in order to see how far the equations of classical general relativity allow us to go. Some encouraging results, which point to a deep connection between the holostar solution and quantum gravity spin-network states, are presented in [28].

## 4 Discussion

The holostar solution appears as an attractive alternative for a black hole. It has a surface temperature, which - measured at spatial infinity - is proportional to the Hawking temperature. Its total number of interior particles is proportional to its proper surface area, which can be interpreted as evidence that the Hawking entropy is of microscopic-statistical origin and the holographic principle is valid in classical general relativity for self gravitating objects of arbitrary size. In contrast to a black hole, the holostar has no event horizon, no trapped surfaces and no central singularity, so there is no information paradox and no breakdown of predictability.

Furthermore, the holostar solution has some potential to serve as an alternative model for a universe with anisotropic negative pressure, without need for a cosmological constant. It also admits microscopic self-gravitating objects with a surface area of roughly the Planck-area and zero gravitating mass.

The remarkable properties of the holostar solution and its high degree of self-consistency should make it an object of considerable interest for future research.

A field of research which presents itself immediately is the the generalization of the holostar solution to the rotating and / or charged case. The derivation of the charged holostar solution is straight forward and will be discussed in a

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<sup>64</sup>According to quantum gravity, a spin-network state with a single spin-0 link has zero area, and thus zero entropy. There are reasons to believe, that a spin-network state with a single spin can be identified with an elementary particle (see [28]). However, a zero area, zero-entropy particle is quite inconceivable. Therefore it is likely, that a fundamental (i.e. not composite) particle with zero spin and charge doesn't exist. Any uncharged spin-zero particle therefore should be composite. For all the elementary particles that are known so far this is actually the case.

follow-up paper [28]. A generalization to the case of a rotating body, including spin (and charge), will be a challenging topic of future research.

An accurate description of the thermodynamic properties of the holostar solution is of considerable interest. In a parallel paper [30] the entropy/area law and the temperature-law for the holostar will be put on a somewhat sounder foundation. The "Hawking temperature" of the universe will be "measured", verifying Hawking's prediction to an accuracy better than 1%.

Another valuable field of research will be the examination, whether the holostar solution can serve as an alternative model for the universe. The holostar solution appears to have the potential to solve many problems of the standard cosmological models, such as the horizon-problem, the cosmological constant problem, the problem of structure formation from the small density perturbations in the CMBR, etc. . It provides an explanation for the numerical value of the baryon to photon ratio  $\eta$ . Even some of the more recent results, such as the missing angular correlation of the CMBR-fluctuations at angular separations larger than  $60^\circ$  or the missing quadrupole moment, which hints at a (small) positive curvature of the universe, are explainable in the context of the holostar space-time. On the other hand, it is far from clear whether the holostar solution will ever be able to explain the observed abundances of the light elements, especially Deuterium and Lithium, in a convincing fashion, such as the standard cosmological model can. Nucleosynthesis in a holostar universe will be a demanding challenge. If the holostar can pass this test, it should open up a considerable field of new and interesting research in the field of cosmology. Quite likely chemical potentials and supersymmetry will play an important role in the holostar universe, not only at high temperatures above the electro-weak unification scale, but also at the time of nucleosynthesis or even at later times.

Lastly, the properties of the membrane, how it is formed, how it is composed and how it manages to maintain its two-dimensional structure will be an interesting research topic, if the holostar turns out to be a realistic alternative for a black hole. The study of anisotropic matter states in high gravitational fields should provide fruitful as well.

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